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APOLLO NOTE NO. C-1
(Task 3, Item III)

H. Engel
24 September 1964

OPTIMUM CORRECTIVE BOOST PROGRAM, II

The computations necessary to find the optimum single corrective boost in the translunar and transearth trajectories are outlined below. Then the results of the computations for the reference translunar trajectory are given and conclusions drawn. The results for the reference transearth trajectory and for varying time translunar and transearth trajectories will appear in subsequent notes.

Flow Diagram for Computations

The program developed for the optimum corrective boost calculations always starts at the lunar end of the flight--at perilune for translunar flight, and at transearth injection for transearth flight. The program contains several alternatives. It may be used to

1. Determine the perigee radius, the location of the injection radius, and the time of flight from injection to perilune for a set of related translunar flights.
2. Determine the perigee radius, the location of perigee, and the time of flight from injection to perigee for a transearth flight.
3. Adjust the perilune conditions for a translunar flight so that the position of the translunar injection point most nearly coincides with a given injection location.
4. Determine the components and the magnitude of the difference between computed perigee location and a desired perigee location for transearth flight.
5. Determine the required corrective boost, its variance, the resultant perilune miss, and the velocity error at perilune for translunar flight for a given injection error.

6. Determine the required corrective boost, its variance, the resultant vacuum perigee miss, the resultant re-entry angle error and its variance for transearth flight for a given injection error.

An outline flow-diagram of the computations is given in Figure 1. The inputs include the basic inertial coordinate components of position and velocity at perilune for translunar flight or at injection for transearth flight. Other inputs are the time of perilune or transearth injection, an indication of whether the flight is transearth or translunar, an indication of which capability of the program is to be employed, the perturbations in perilune or transearth injection conditions that are to be used, the injection radius and desired injection location for translunar flight, the translunar or transearth injection error, and the error in the performance of the corrective boost. An ephemeris of the Moon in basic inertial coordinates may also be an input to the program, or the ephemeris may be calculated as though the Moon were in a circular orbit in the plane of the Earth's equator.

Using the basic inertial coordinate components of position and velocity of perilune or of transearth injection, the rotation matrix G that transforms a state vector of six components (3 position, 3 velocity) from the basic inertial coordinate system to the plane of a Moon-centered conic is calculated. This new coordinate system is the x'_m, y'_m, z'_m coordinate system. If $\dot{z}'$ is to be varied, this is done and a new matrix G computed. The parameters a_1, a_4 and a_5 are computed (See Apollo Note No. 83, or Final Report on Capabilities of MSFN for Apollo Guidance and Navigation for definitions of these and other quantities). The magnitude of the velocity and its angle to the horizontal are computed; if these are to be varied (ΔV, $\Delta \psi$) the perturbations are added and a_4 and a_5 recomputed. Next, the location of the intersection of the trajectory with the lunar sphere of influence (LSOI) the velocity at this point, and the time of intersection are computed. An ephemeris look-up or

Given: X_o, Y_o, Z_o, $\dot{X}_o$, $\dot{Y}_o$, $\dot{Z}_o$, T_o, Translunar or Transearth, Iteration, Boost, $(\Delta X''_o, \Delta \dot{X}''_o)$, σ_{ξ}, ΔV, $\Delta\psi$, $\Delta \dot{Z}'$, R_I, Ephemeris

1 Find $\hat{x}'$, $\hat{y}'$, $\hat{z}'$, G using $\Delta \dot{z}'$

Applying ΔV, $\Delta\psi$

Find a_{1M}, a_{4M}, a_{5M}

Intersection with LSOI

Time of intersection

Perform ephemeris look-up

Find $\hat{x}'_E$, $\hat{y}'_E$, $\hat{z}'_E$, G_1

a_{1E}, a_{4E}, a_{5E}, R_{pE}

Translunar?

yes

no

R_{pE} R_I ?

Find time of flight from injection to vacuum perigee

no

yes

Go to next case

Find time of flight from injection to perilune

Optimum boost calculation?

yes

no

Translunar iteration?

yes

no

Obtain $Q'_{L,M}$, $Q'_{inj,L}$ from Program A

Transearth iteration?

Find θ_E, G_2

yes

no

Go to next case

$$Q'_{peri,\,inj} = \left[G_2 \, Q'_{inj,\,L} \, G_1 G^T Q'_{L,\,M} \right]^{-1}$$

Optimum boost?

yes

no

Figure 1.

Find $Q'_{t, inj}$, $\text{cov } \Delta\hat{x}(t)$

$Q_{p, t}$, $\text{cov } \Delta\hat{x}_p$

$\left[E\,(p^{*T}, P^*)\right]^{1/2}$

$\left|E\,(\Delta v)\right|$

$\left[\text{Var}\,(\Delta v)\right]^{1/2}$

$E\,[V^*]$

Find $X''_{inj, given}$, $\Delta X''_{inj}$

Translunar?

yes

Find ΔV_{peri}

Find new

$\dot{X}_o$, $\dot{Y}_o$, $\dot{Z}_o$

Go to 1

no

Find $\left|\Delta X''_{inj}\right|$

Go to next case

Figure 1.
(continued)

computation yields the position and velocity of the Moon at this time, and these are used to determine the position and velocity of the vehicle with respect to the Earth at this time. The rotation matrix G_1, that transforms a state vector from the basic inertial coordinate system to the plane of the Earth-centered conic is computed. This new coordinate system is the x'_E, y'_E, z'_E system, and the orbit parameters at the LSOI in this system are a_{1E}, a_{4E} and a_{5E}. The perigee radius is computed. If this is a transearth flight, the time from injection to perigee is computed. If this is a translunar flight, then the perigee radius R_{pE} is compared with the desired injection radius R_I. If $R_{pE} > R_I$, the next perturbation of lunar conditions is started. If $R_{pE} \leq R_I$, the time of flight from injection to perilune is calculated.

At this point several decisions are made in the program. If the optimum boost calculations are to be performed, or if this is a translunar iteration (i.e., the injection location is to be adjusted to coincide with a give injection point) then program A of the Bissett-Berman error analysis program is employed to find the transition matrices $Q'_{L,M}$ and $Q'_{inj,L}$. $Q'_{L,M}$ gives the change in a state vector at the LSOI in terms of the change in a state vector at perilune; both state vectors are in x'_M, y'_M, z'_M coordinates. $Q'_{inj,L}$ gives the change in a state vector at translunar injection or perigee in terms of a change in the state vector at the LSOI; both state vectors are in x'_E, y'_E, z'_E coordinates. The rotation matrix G_2 that transforms a state vector from x'_E, y'_E, z'_E coordinates to x''_E, y''_E, z''_E coordinates is determined, where x''_E is through translunar injection or perigee, and y''_E is in the plane of motion. The transition matrix $Q'_{peri,inj}$ which gives the change in a state vector at perilune or transearth injection in terms of a change in the state vector at translunar injection or perigee is computed.

If this is an optimum boost calculation, then programs A and B of the Bissett-Berman error analysis program are used to obtain the covariance matrix of the state vector at various times after injection, together with the corresponding transition matrix $Q'_{t,\,inj}$. Then for each time the transition matrix $Q_{perigee,\,t}$ or $Q_{perilune,\,t}$ is found. Using the injection error $(\Delta X_o'', \Delta \dot{X}_o'')$ the mean square value of the miss at perilune or vacuum perigee is computed, together with the expected value of the corrective boost, and its variance which depends on the error σ_ε in the execution of the commanded boost. The resultant velocity difference at perilune is computed for translunar flight. See Apollo Note No. 239. For transearth flight the expected error in re-entry angle and its variance are computed.

Returning now to the point in the program just after determination of the time of flight, if this is not an optimum boost calculation or a translunar iteration or a transearth iteration, the computations start again with the next perturbation.

If the computations are a transearth iteration--a misnomer carried over from an earlier version of the program, and now indicating only that the difference between the computed perigee position and the desired perigee position is to be computed--or if the computations are a translunar iteration, the position of the desired perigee or transearth injection point are found in x''_E, y''_E, z''_E coordinates, and the difference $\Delta X''$ between the computed and desired values computed. If this is a transearth iteration, the computations start again with the next case; if it is a translunar iteration, then the necessary change in conditions at perilune to adjust the injection location are computed from $Q'_{peri,\,inj}\,\Delta X'$ and the computations repeated with the new perilune conditions.

In the program, the Moon-centered orbits are currently restricted to hyperbolas, and the Earth-centered orbits to ellipses. There are many printouts in the various phases of the program so that results may be graphed.

Translunar Reference Trajectory

The optimum midcourse correction study is to be performed for an AMPTF reference mission, and for flights of longer duration, up to 110 hours. The injection and perilune state vectors for the reference mission have been specified for patched conic approximations as:

	X (n. mi.)	Y (n. mi.)	Z (n. mi.)	
Translunar Cutoff	-1979.0874	2328.1528	1924.0770	
Perilune	331.31047	- 886.01090	- 399.80462	
	$\dot{X}$ (ft/sec.)	$\dot{Y}$ (ft/sec.)	$\dot{Z}$ (ft/sec.)	Time (GMT)
Translunar Cutoff	-27937.154	-21544.653	4916.3660	260 days 15.9206 hrs.
Perilune	-7901.2016	- 1833.1348	-2484.8179	263 days 5.11897 hrs.

for the year 1969 and using basic body-centered inertial equatorial coordinate systems with X axes directed toward the mean equinox of date and Z axes directed parallel to the North direction along the Earth's axis. Time is referred to 00 hours 31 December 1968. A suitable lunar ephemeris has been supplied by MSC.

The program described in the preceding section has been applied, resulting in a set of patched conics for the reference mission differing slightly from that specified above because we are employing slightly different values for the physical constants and for the radius of the LSOI.

For comparison,

	X (n. mi.)	Y (n. mi.)	Z (n. mi.)	
Translunar Cutoff	-2161.929	2141.803	1943.621	
Perilune	331.31047	-886.01090	-399.80426	
	$\dot{X}$ (ft/sec.)	$\dot{Y}$ (ft/sec.)	$\dot{Z}$ (ft/sec.)	Time of Flight (hrs)
Translunar Cutoff	-27484.67	-22346.15	4096.7944	
Perilune	-7841.943	-1856.283	-2476.036	61.28

Visibility of the vehicle from various MSFN stations for the injection conditions utilized are shown in Figure 2. In determining the covariance matrix of the state vector, range only measurements from Madrid, Ascension and Antigua have been employed, with an a priori range bias uncertainty of 20 m from each station and a random error on each range measurement of 15 m with measurements from all three stations obtained once each minute, starting 10 minutes after injection. No other a priori orbit data is used. Now, we know that the implementation of the S-band transponders aboard the vehicle does not permit simultaneous ranging from multiple MSFN stations. Instead, if the range mode is used, the stations must be time-shared. This will result in increasing the standard deviations of uncertainty in position and velocity by a factor of about $\sqrt{3}$, but, as will be seen, this will not significantly affect the conclusions drawn concerning the optimum corrective boost scheduling. Further, in a subsequent Bissett-Berman

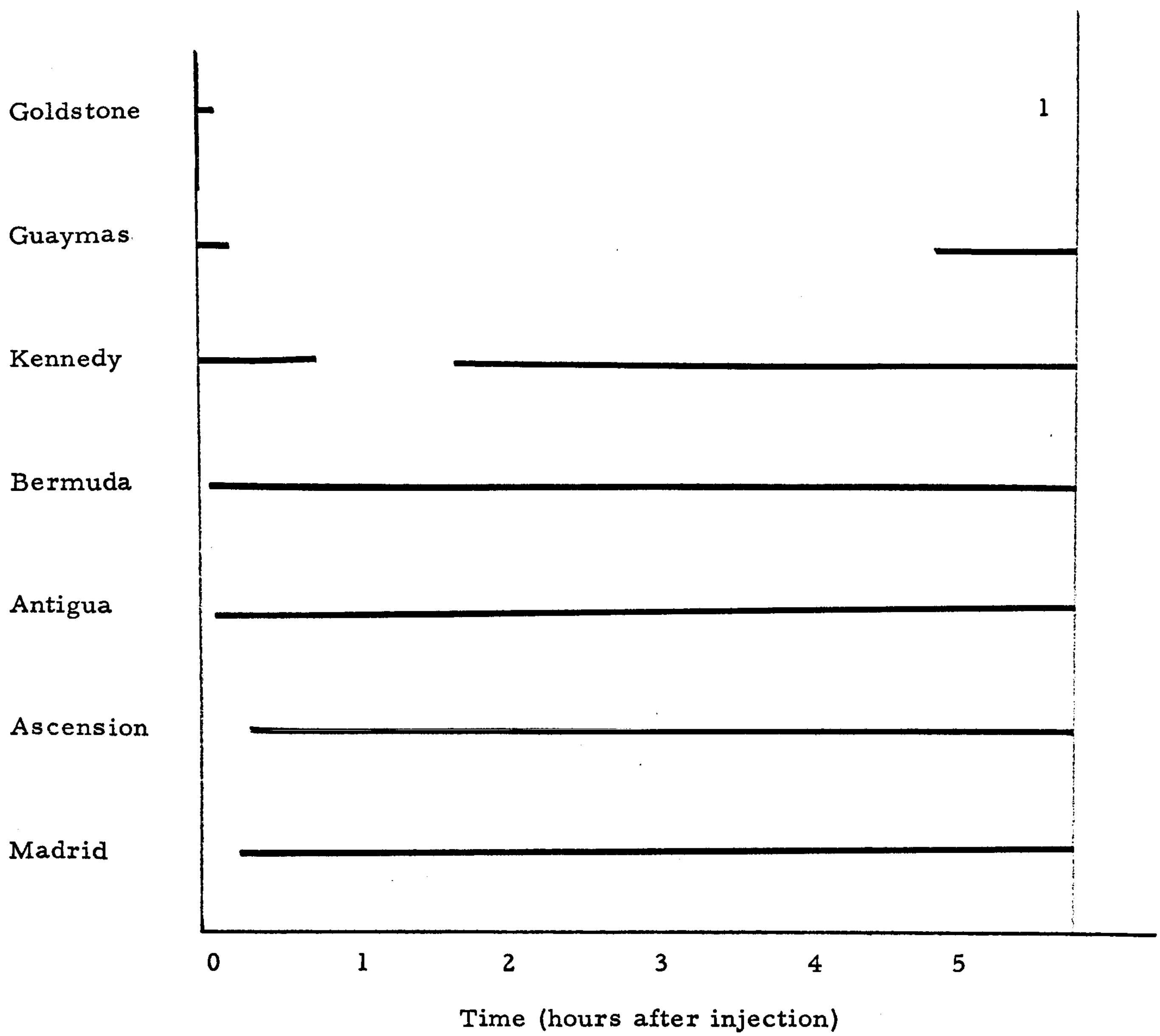
Goldstone
Guaymas
Kennedy
Bermuda
Antigua
Ascension
Madrid
1
0
1
2
3
4
5
Time (hours after injection)

Figure 2.

Apollo Note to follow shortly, on varying time translunar trajectories, the computations will be performed not only for range measurements, but also for Doppler measurements that do permit simultaneous measurements from several stations. Still further, it will be shown in that note that the covariance matrices for the Doppler measurements are smaller than for the range measurements; as a result the conclusions drawn in the present note will be even more strongly evident if Doppler data is used.

Reasonable assumptions concerning the characteristics of the on-board guidance system lead to injection velocity errors with 3 σ values of 11.3 m/sec. in the radial and out-of-plane directions and 1.5 m/sec. in the tangential direction. The largest components of the position error at translunar injection are radial and out-of-plane and have 3 σ values of 1800 meters.

Figure 3 indicates what the RMS value of miss will be at the nominal time of perilune after a single corrective boost, and also the expected value of this boost, both as functions of the time at which the corrective boost is applied. These results are for injection velocity errors of

11.5 m/sec. radially, 1.5 m/sec.

tangentially, and 11.6 m/sec. in the orthogonal direction, as derived above.

The RMS value of perilune miss is a function of the error in the execution of the commanded corrective boost. In fact, it is a very strong function of the execution error, the major portion of the miss resulting from a corrective boost any time later than 40 minutes after injection being due to the error in the execution of the commanded corrective boost.

In order to make this analysis simpler the errors in the execution of the commanded corrective boost have been assumed Gaussian and distributed isotropically. The standard deviation of the boost execution error in each direction is proportional to the total boost, so that if the commanded boost (i.e., the expected value of corrective boost) is 20 m/sec. and the standard deviation σ_ε of the execution error is specified as 10^{-3}, then the RMS value of each component of the error in executed boost resulting is 0.02 m/sec ($=20 \times 10^{-3}$). This, incidentally, corresponds to an alignment error of 1 mil (0.057°) and a cut-off error of 0.02 m/sec. For a boost of 20 m/sec., σ_ε equal to 10^{-2} would correspond to an alignment error of 10 mils (0.57°) and a cut-off error of 0.2 m/sec.

The miss resulting from a zero boost execution error is plotted in Figure 3 for comparison purposes. It represents how well the miss can be predicted, before the corrective boost, from the radar data. It can be seen that as early as one hour after injection the perilune miss can be predicted with an RMS error of only 4600 m ($\sim$ 2 n. mi.), and that the accuracy of the perilune miss prediction continues to improve rapidly with time. Madrid, Ascension and Antigua can not see the vehicle for the entire 28 hours after injection. They can see it for more than 5 hours after injection. Nevertheless, the remainder of the miss curve for σ_ε equal to zero has been computed as though the same three stations were used. This made the computations easier, and actually results in a slightly conservative estimate (i. e., too large) of the uncertainty in perilune miss, or of the actual miss after a corrective boost, because those stations on the side of the Earth towards the vehicle, being closer to the vehicle, will have a more favorable geometry for determination of the orbit.

For σ_ε equal to 10^{-3} or 10^{-2}, most of the miss after the first corrective boost is due only to the execution error. For instance, considering a corrective boost at 4 hours after injection, the miss at perilune would be 2400 meters if the boost could be executed perfectly, 6800 meters if σ_ε is 10^{-3}, and 63,000 meters if σ_ε is 10^{-2}.

It will be observed that for corrective boost times greater than 6 hours after injection the miss resulting from σ_ε equal to 10^{-2} is very nearly exactly 10 times the miss resulting from σ_ε equal to 10^{-3}, indicating that both misses depend almost entirely on σ_ε.

An alignment accuracy of 1 mil during corrective boost is not unreasonable. Consequently it appears to be very worthwhile to apply the first corrective boost entirely with the attitude control jets, or to start the boost with the main motor and complete it with the attitude jets in order to avoid the velocity error resulting from tail-off of the main motor. If this is done, the RMS misses represented by the curve for σ_ε equal to 10^{-3} can be achieved -- at least a factor of 10 better than could be accomplished using the main motor.

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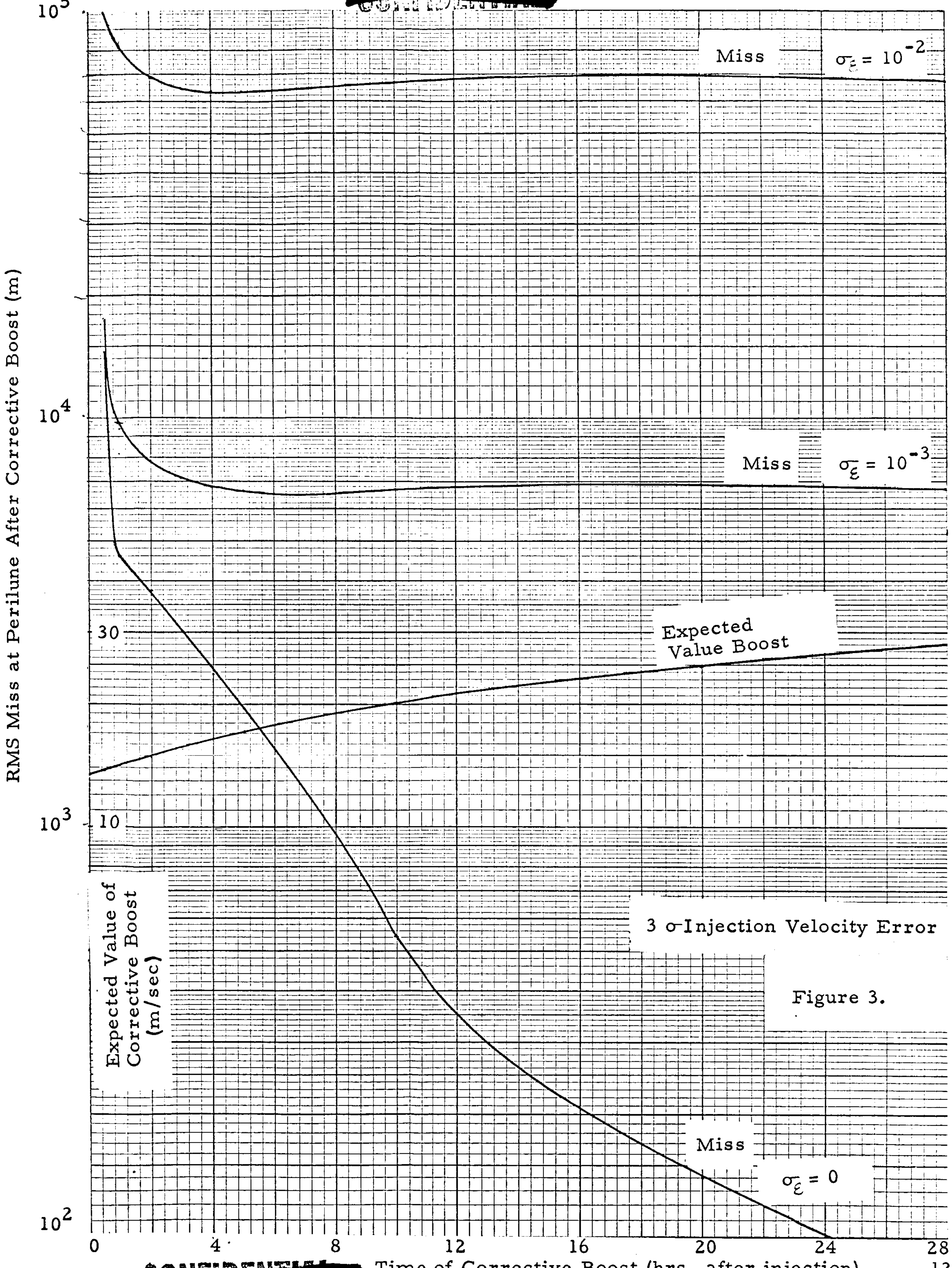


Figure 3.

[redacted]

Another important point to be observed in Figure 3 is that the miss curves are very flat, so that the miss is not strongly dependent upon the time of corrective boost. The expected value of corrective boost increases with time, and is independent of σ_ε . As a consequence, it is desirable to perform the corrective boost as early as possible--certainly as early as 4 hours after injection.

It can, in fact, be shown that for minimum total corrective boost cost the first corrective boost should be performed even earlier, that is, at 1 hour after injection, allowing only enough time for transportation and docking of the LEM and jettisoning of the SIV-B. This can be seen as follows: For σ_ε equal to 10^{-3} a corrective boost 4 hours after injection has an expected value of 16.5 m/sec. and leaves a miss of 6800 meters to be eliminated by a subsequent boost. This miss of 6800 meters results from the orbit uncertainty and the boost execution error, which together result in a standard deviation of just 0.024 m/sec., in the error in velocity after the first corrective boost (see Figure 4) indicating that the cost of eliminating the remaining miss, if it could be done immediately, would be of the order of 0.024 m/sec. Of course this error can not be eliminated immediately, but even if the cost grew at the same rate as the cost of an initial velocity error at injection, the cost of eliminating this remaining error with a second boost 28 hours after injection would be only 0.041 m/sec. (=0.024 x 28/16.5). The total corrective boost cost would then be 16.5 + 0.041 m/sec.

For comparison, consider the first corrective boost performed 1 hour after injection. The first corrective boost is 14.1 m/sec., the resultant miss with σ_ε equal to 10^{-2} is 9700 meters, and the boost required to eliminate this resultant miss immediately is of the order of 0.028 m/sec. Waiting until 28 hours after injection, the cost of the second boost is of the order of 0.058 m/sec. (=0.028 x 28/14.1) The total cost for both corrective boosts is then 14.1 + 0.058 m/sec. for the first corrective boost at 1 hour after injection, versus a total cost of 16.5 + 0.041 m/sec. for the first corrective boost 4 hours after injection, so that it is apparent that the corrective boost should be performed as early as practicable, for this particular injection error.

Figure 4, in addition to showing the standard deviation of the error in corrective boost for σ_ξ equal to 0, 10^{-3} and 10^{-2}, also shows the magnitudes of the expected values of the radial, tangential and out-of-plane components of error in velocity at perilune after the corrective boost. The out-of-plane component is the largest for all the times of corrective boost considered; the cost of compensating for this or for the radial component of error in velocity during the perilune deboost is negligible, since these errors are orthogonal to the deboost velocity. The tangential component may increase the cost of the perilune deboost, but this cost is small since the magnitude of the tangential component is less than 0.5 m/sec. for any corrective boost time less than 10 hours after injection.

Reference Trajectory Parametric Study

The discussion of the previous section and Figures 3 and 4 corresponds to a particular injection error. The necessary boosts and the resultant misses at perilune, however, are strong functions of the kinds of injection errors. This section considers errors in position and velocity at injection parametrically. Radial, tangential and orthogonal (out-of-plane) position errors are considered separately. Velocity errors along the velocity vector, perpendicular to it in-plane, and orthogonal (out-of-plane) are considered separately.

Figure 5 shows the RMS perilune misses resulting after one corrective boost for injection position errors of 5000 m radially tangentially and out-of-plane, all for σ_ξ equal to 10^{-3}. The radial position error results in the largest miss. Figure 6 compares the misses after the first corrective boost for radial injection position errors of 5000 m with σ_ξ equal to 10^{-3} and equal to 10^{-2}. The misses for σ_ξ equal to 10^{-2} are ten times larger, indicating that the miss is due primarily to the error in execution of the commanded corrective boost for both values of σ_ξ .

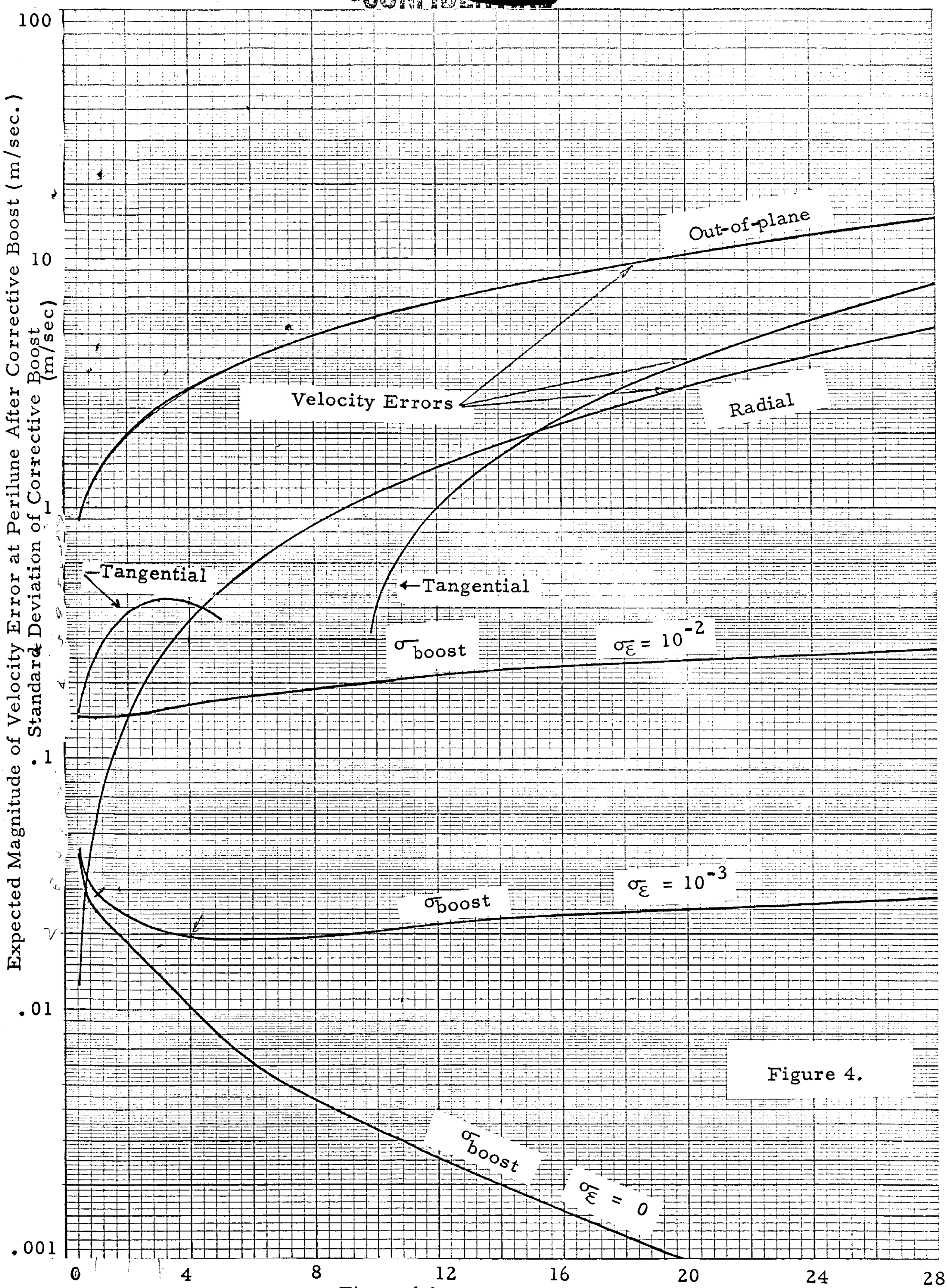
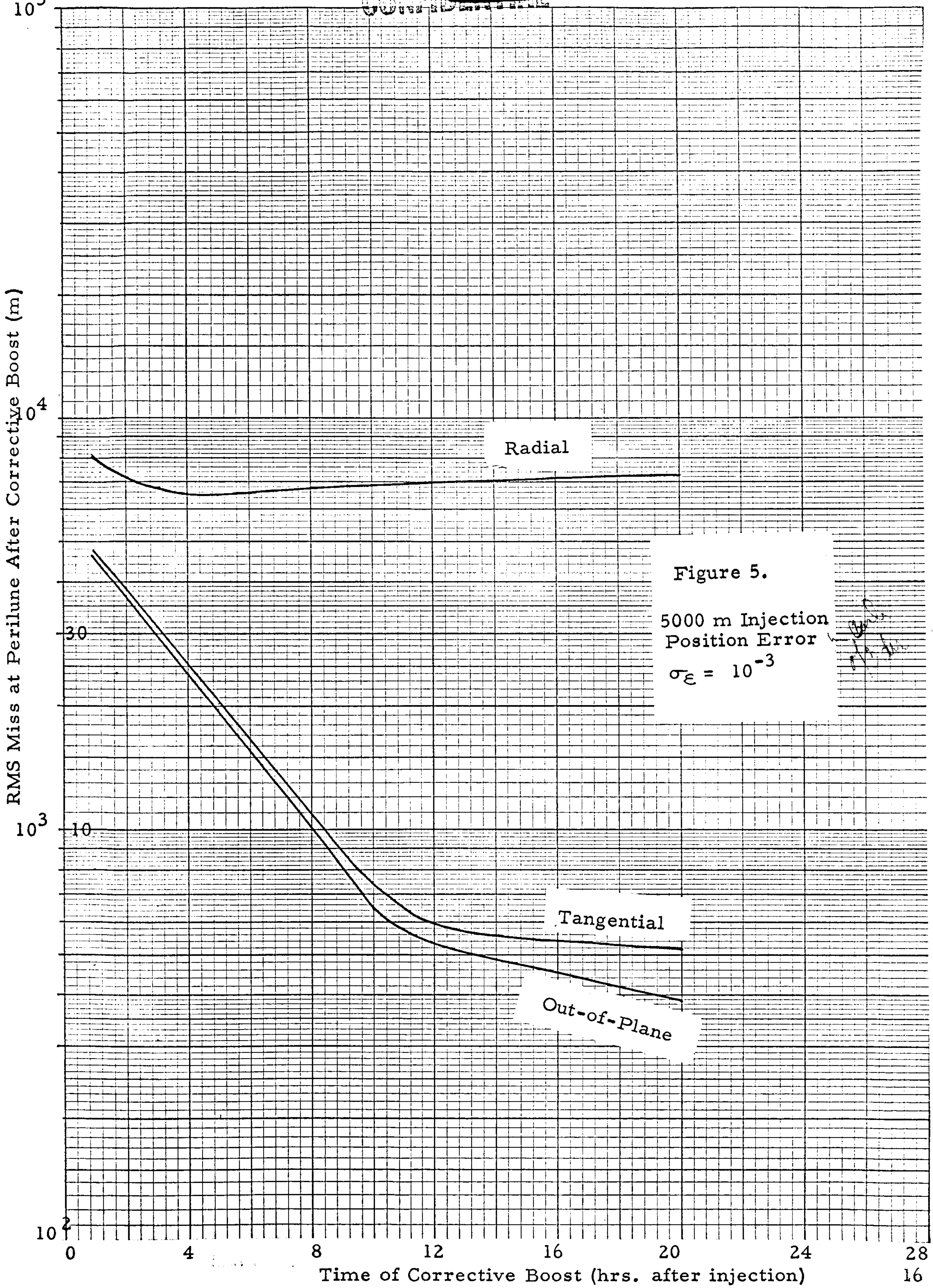

100
10
1
.1
.01
.001
Expected Magnitude of Velocity Error at Perilune After Corrective Boost (m/sec.)
Standard Deviation of Corrective Boost (m/sec)
Out-of-plane
Velocity Errors
Radial
Tangential
←Tangential
σ boost
$\sigma_\varepsilon = 10^{-2}$
σ boost
$\sigma_\varepsilon = 10^{-3}$
σ boost
$\sigma_\varepsilon = 0$
0
4
8
12
16
20
24
28
Time of Corrective Boost (hrs. after injection)

Figure 4.

[redacted]



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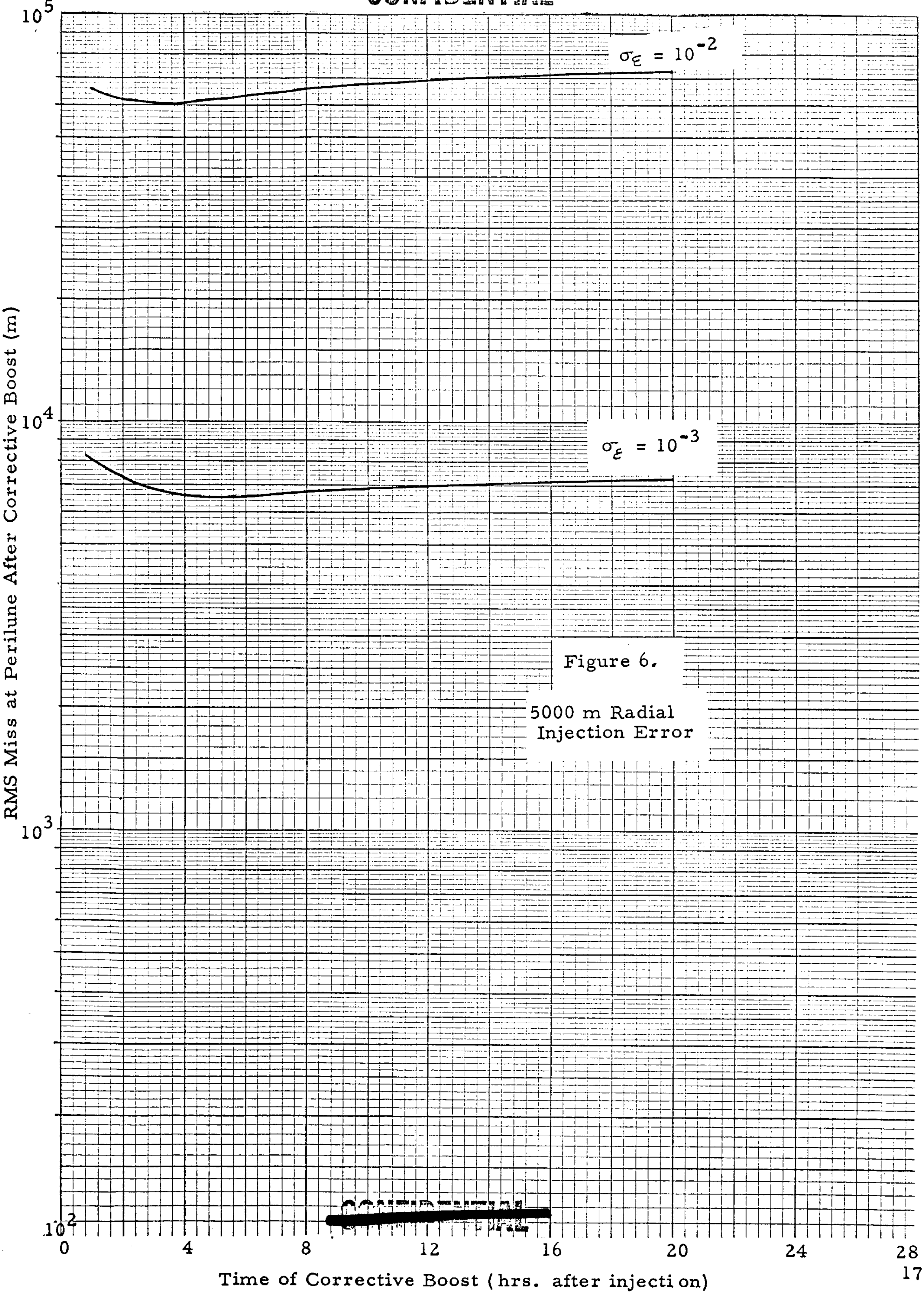

Figure 6.

5000 m Radial Injection Error

Figure 7 shows the expected values of corrective boost for radial, tangential and orthogonal injection position errors of 5000 m. These are independent of σ_ξ. The boost costs to compensate for tangential and orthogonal injection errors are so nearly alike as to appear as a single curve in the figure. The fact that the corrective boost costs decrease for a while with increasing time for tangential and orthogonal injection position errors indicates that under some circumstances it might be worthwhile to delay the corrective boost. This will also appear to be the case for perpendicular (in-plane) and orthogonal velocity injection errors. Nevertheless, it will still generally be the best policy to perform the corrective boost as early as possible; this is so because the magnitude of the corrective boosts to compensate for these components of errors will generally be small compared to those required to compensate for radial injection position errors or for injection velocity errors along the velocity vector.

Figures 8, 9 and 10 indicates the misses following the first corrective boost for injection velocity errors parallel to the velocity vector, perpendicular to it in-plane, and orthogonal to it out-of-plane, respectively, all for σ_ξ equal to 10^{-3}. In each case injection velocity errors of 3, 10 and 30 m/sec. have been considered. As stated previously, the misses after the first corrective boost are largest for injection velocity errors along the velocity vector.

For injection velocity errors along the velocity vector the misses after the first corrective boost vary in proportion to the injection error, being primarily dependent on the error in the execution of the corrective boost. For injection velocity errors in the other two directions this sort of behavior is not evident until long after injection, because the magnitude of the corrective boost is much smaller and the boost execution error does not become large compared to the execution error for a long time.

Since the miss after the first corrective boost is largest for injection velocity errors along the velocity vector, this miss has been

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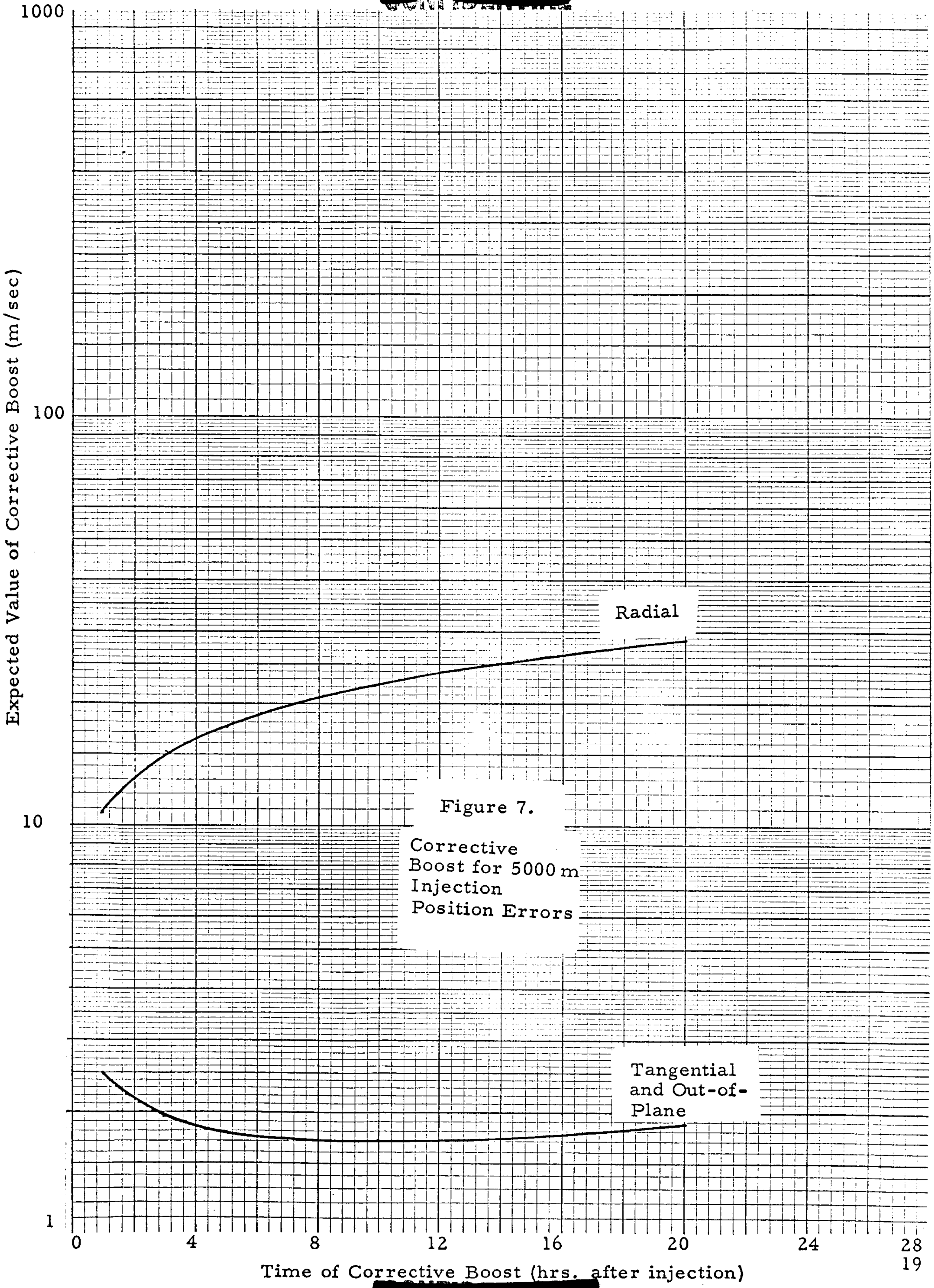

Figure 7.

Corrective Boost for 5000 m Injection Position Errors

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[redacted]

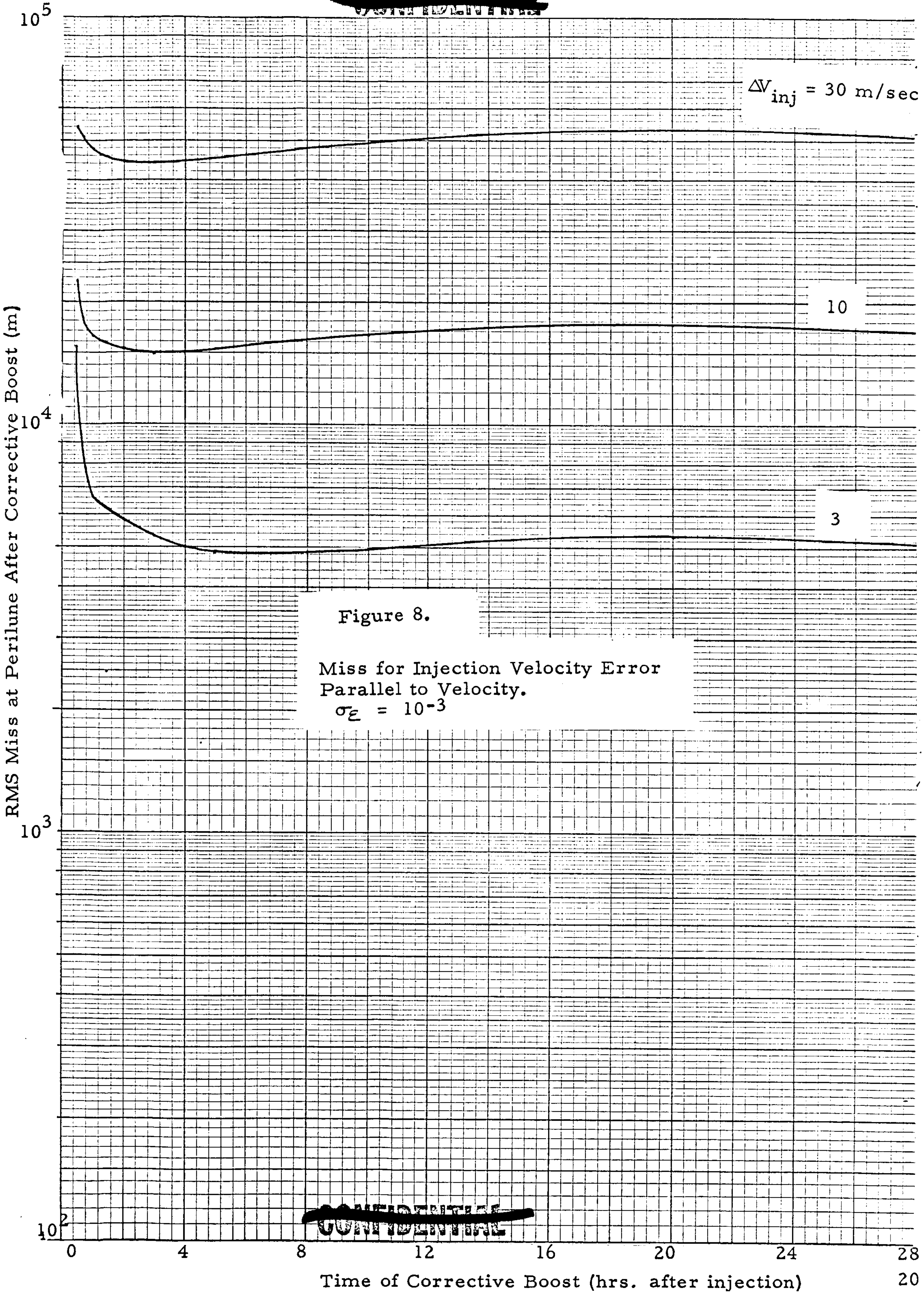

Figure 8.

Miss for Injection Velocity Error Parallel to Velocity.
$\sigma_{\varepsilon} = 10^{-3}$

[redacted]

[redacted]

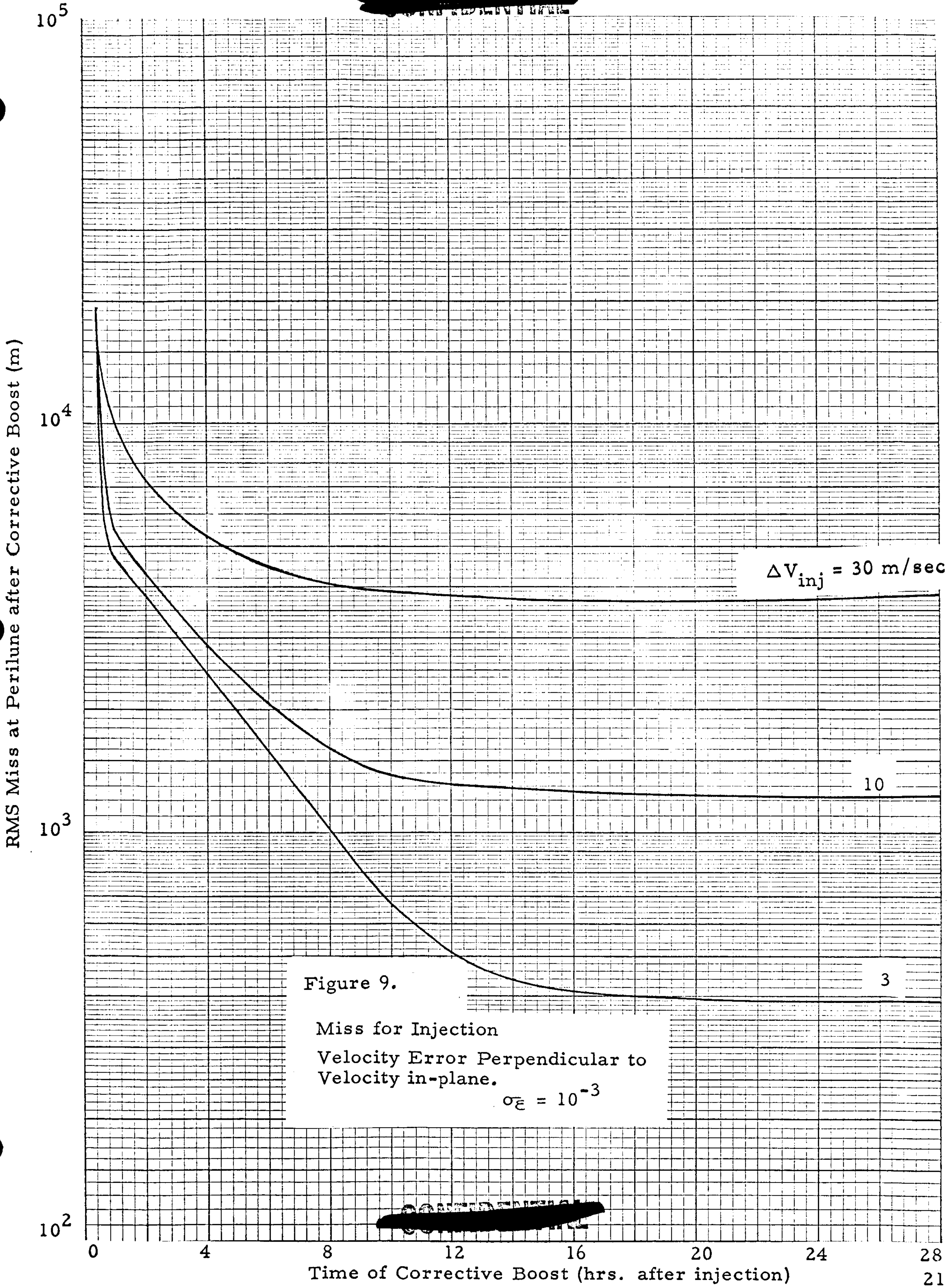

Figure 9.

Miss for Injection Velocity Error Perpendicular to Velocity in-plane. $\sigma_\varepsilon = 10^{-3}$

[redacted]

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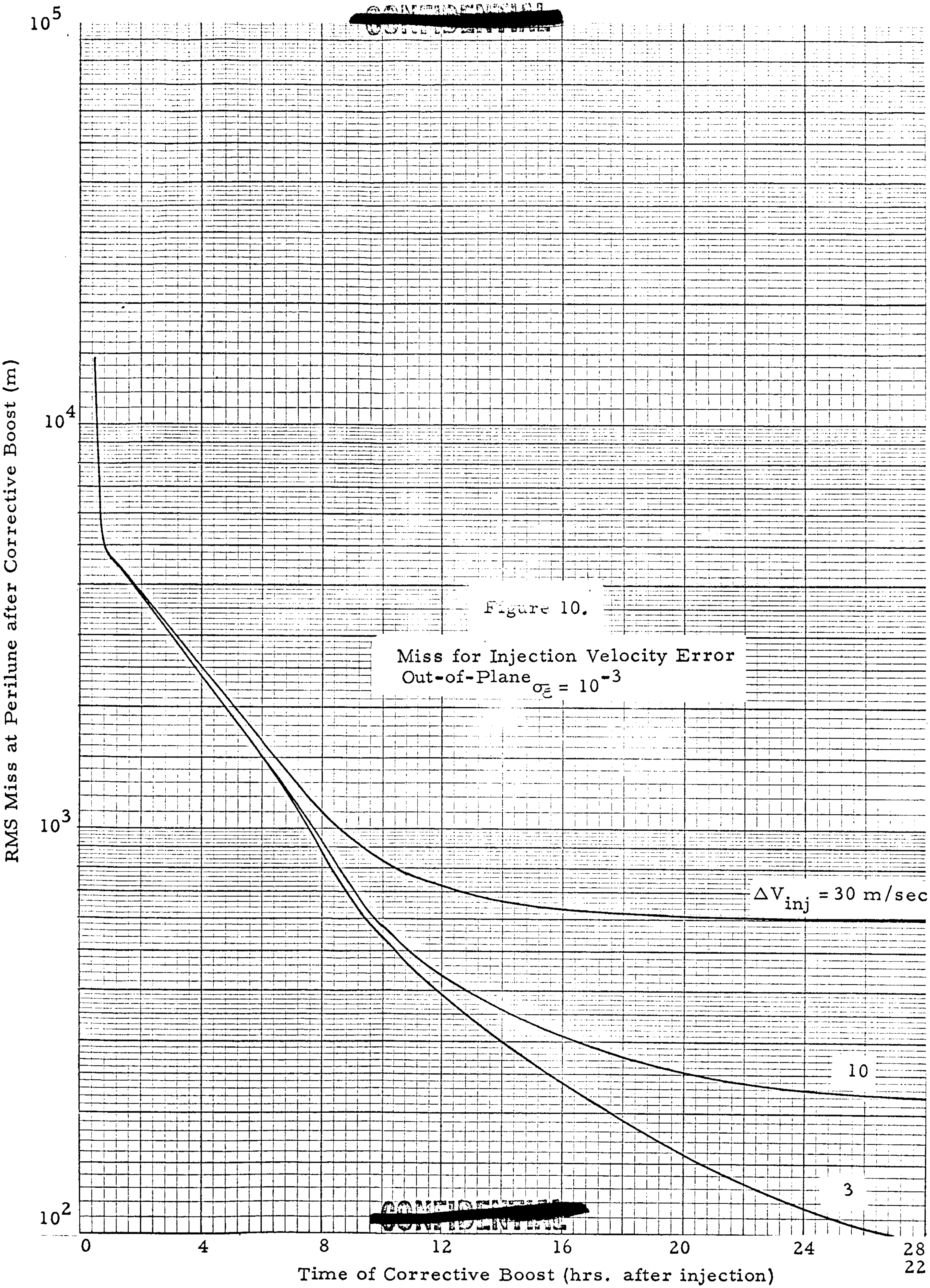

Figure 10.

Miss for Injection Velocity Error Out-of-Plane $\sigma_{\xi} = 10^{-3}$

[redacted]

chosen for a display of the effect changing parameters. Figure 11 shows the resultant miss for injection velocity errors of 3, 10 and 30 m/sec. along the velocity vector, and for σ_ε equal to 10^{-3} and 10^{-2}. For these ranges of parameters the resultant miss is found to be proportional to the injection error and to σ_ε.

The corrective boosts corresponding to Figures 8, 9 and 10 are shown in Figures 12, 13 and 14. They are proportional to the injection velocity errors and independent of σ_ε. The standard deviation of the corrective boosts from the expected values are small. They are shown in Figure 15 for injection errors parallel to the velocity vector, the case in which they are largest, for σ_ε equal to 10^{-3} and 10^{-2} and for injection velocity errors up to 30 m/sec. In general, for corrective boosts performed early, as recommended, they are small.

The resultant errors in velocity at perilune are also worst for injection velocity errors parallel to the velocity vector. The components of these are plotted in Figure 16 for an insertion velocity error of 10 m/sec. They are independent of σ_ε, but proportional to the errors in injection velocity.

Conclusions

For translunar flights of the order of 70 hours duration there is no difficult optimum boost scheduling problem to be solved. The first corrective boost is made as soon as physically possible. The boost cost of any second or third corrective boost is negligible.

The magnitude of the first corrective boost is highly sensitive to the direction of the injection error, being largest for position errors in the radial direction and velocity errors parallel to the velocity.

The magnitude of the miss after the first corrective boost is dependent primarily upon the error with which the commanded boost is executed, rather than on the ability of the MSFN to determine the orbit--for the physical model employed here. Even with large errors

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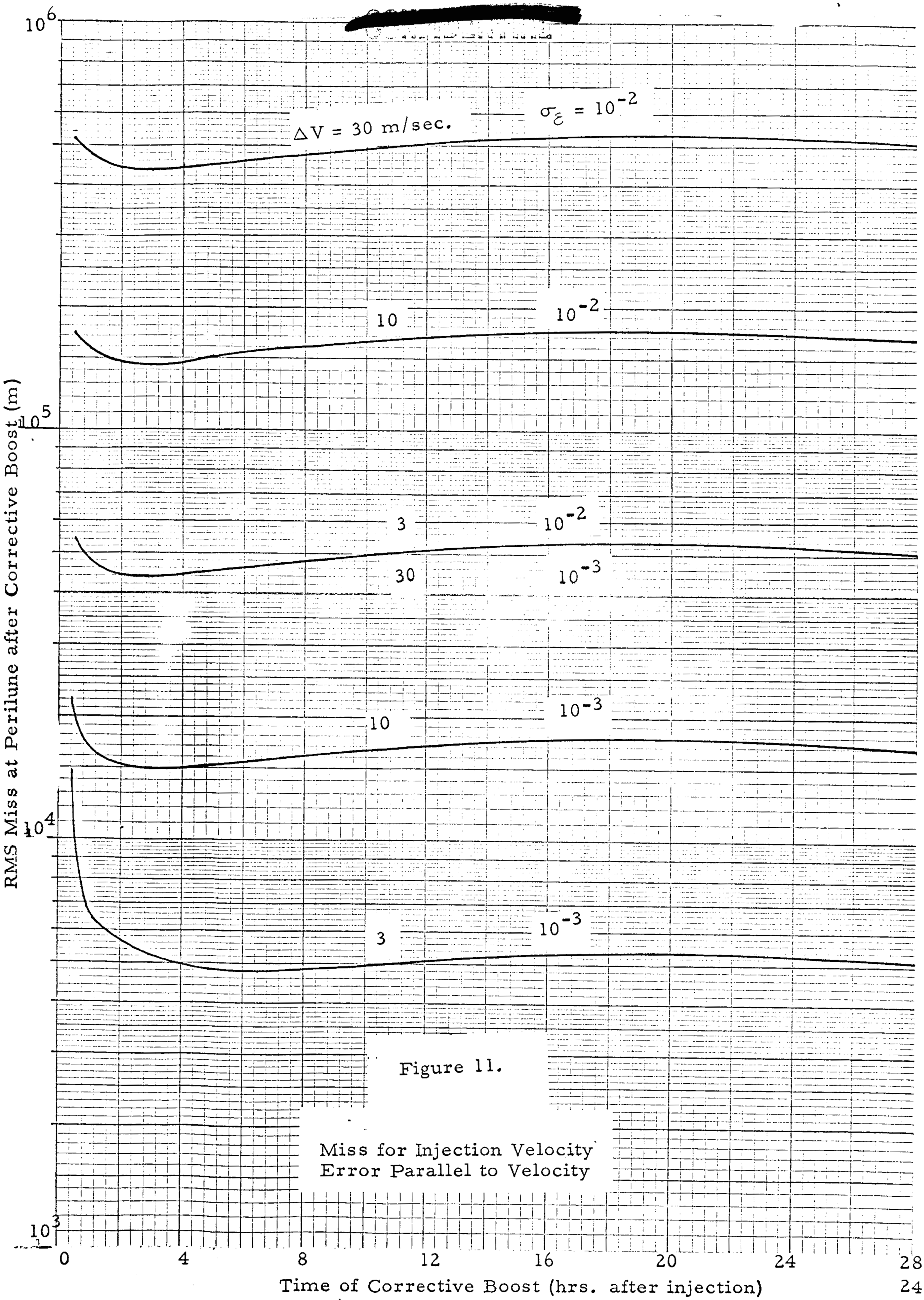

Figure 11.

Miss for Injection Velocity Error Parallel to Velocity

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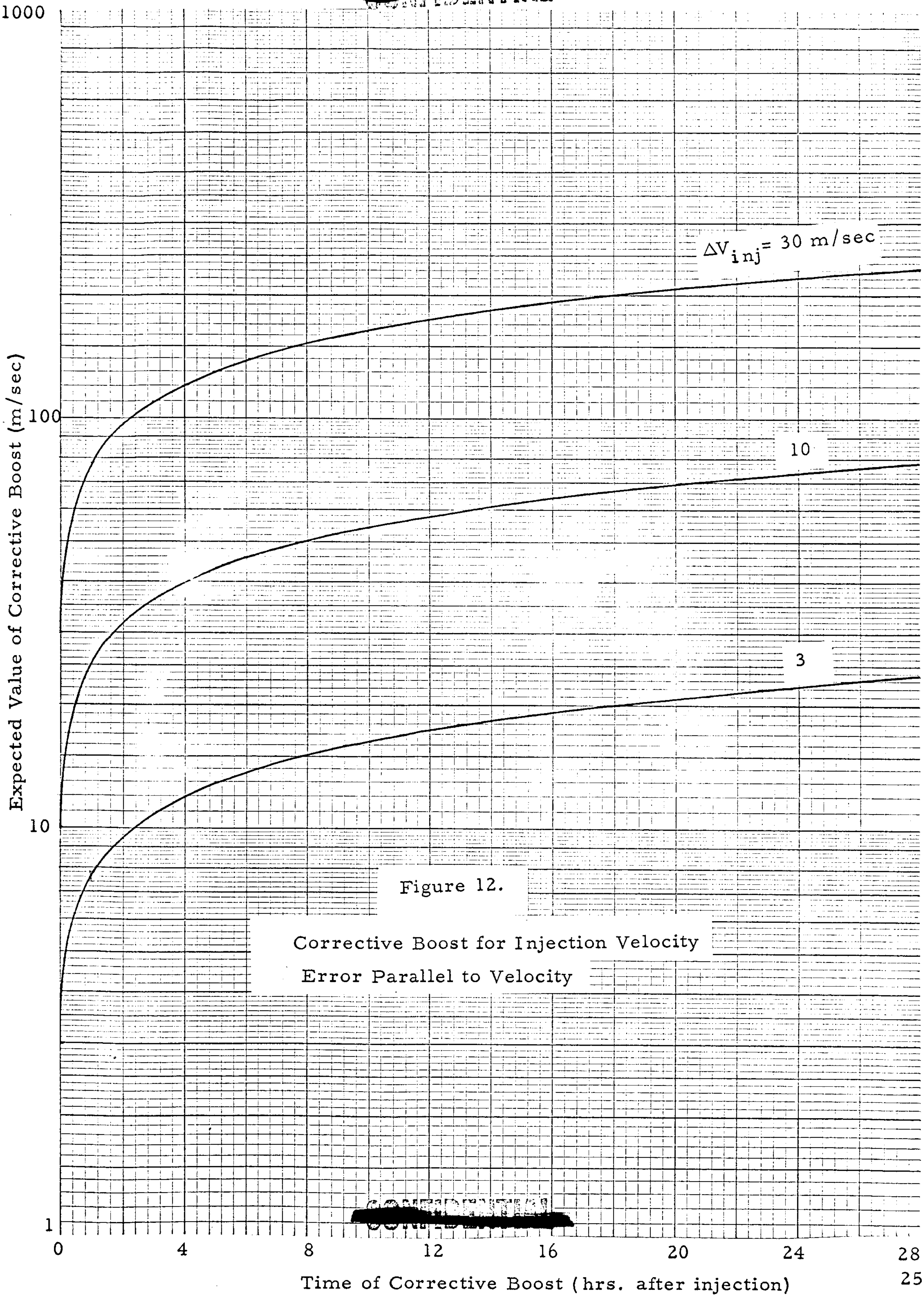

Figure 12.

Corrective Boost for Injection Velocity Error Parallel to Velocity

[redacted]

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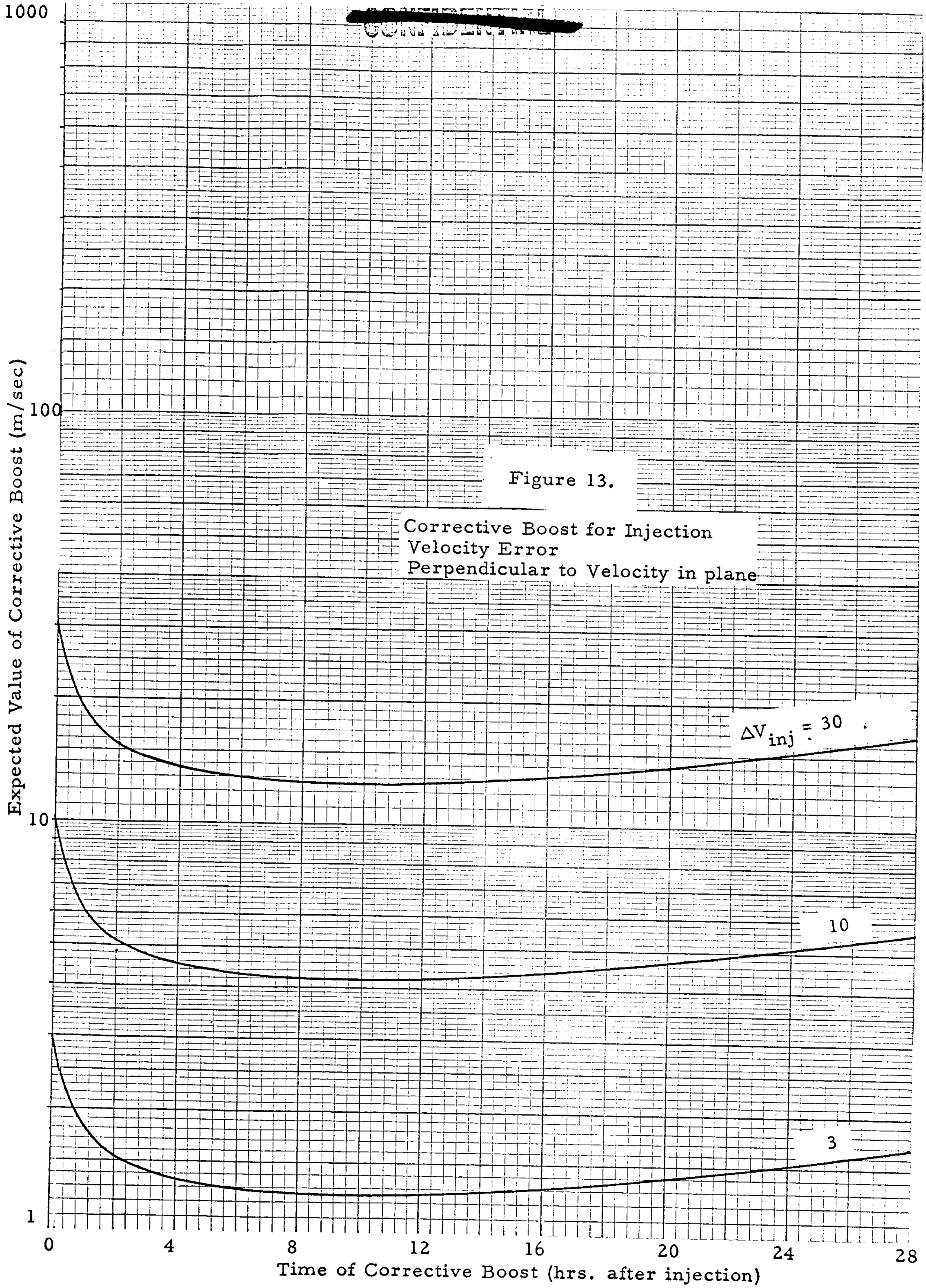

Figure 13.

Corrective Boost for Injection Velocity Error Perpendicular to Velocity in plane

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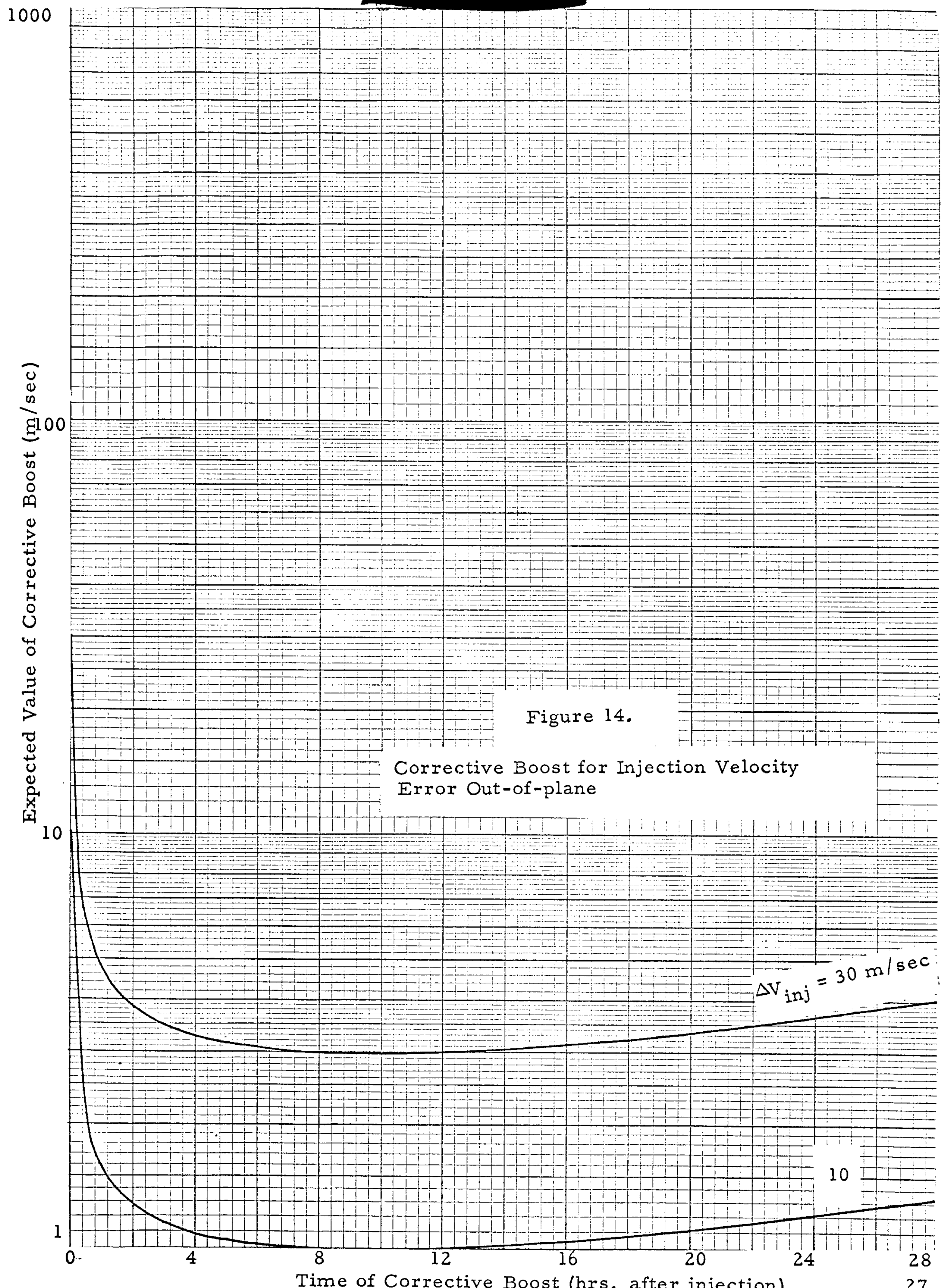

Figure 14.

Corrective Boost for Injection Velocity Error Out-of-plane

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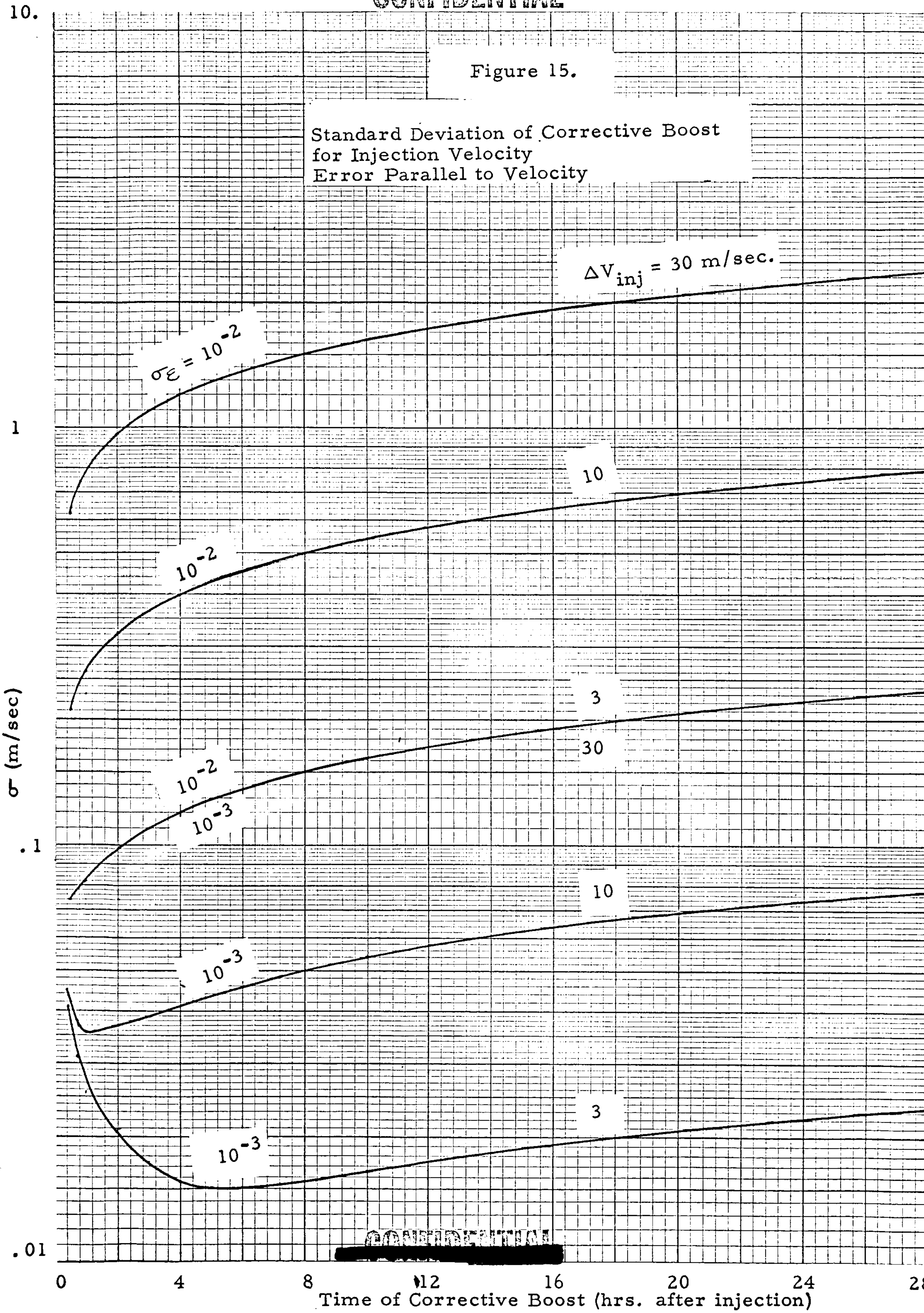

Figure 15.

Standard Deviation of Corrective Boost for Injection Velocity Error Parallel to Velocity

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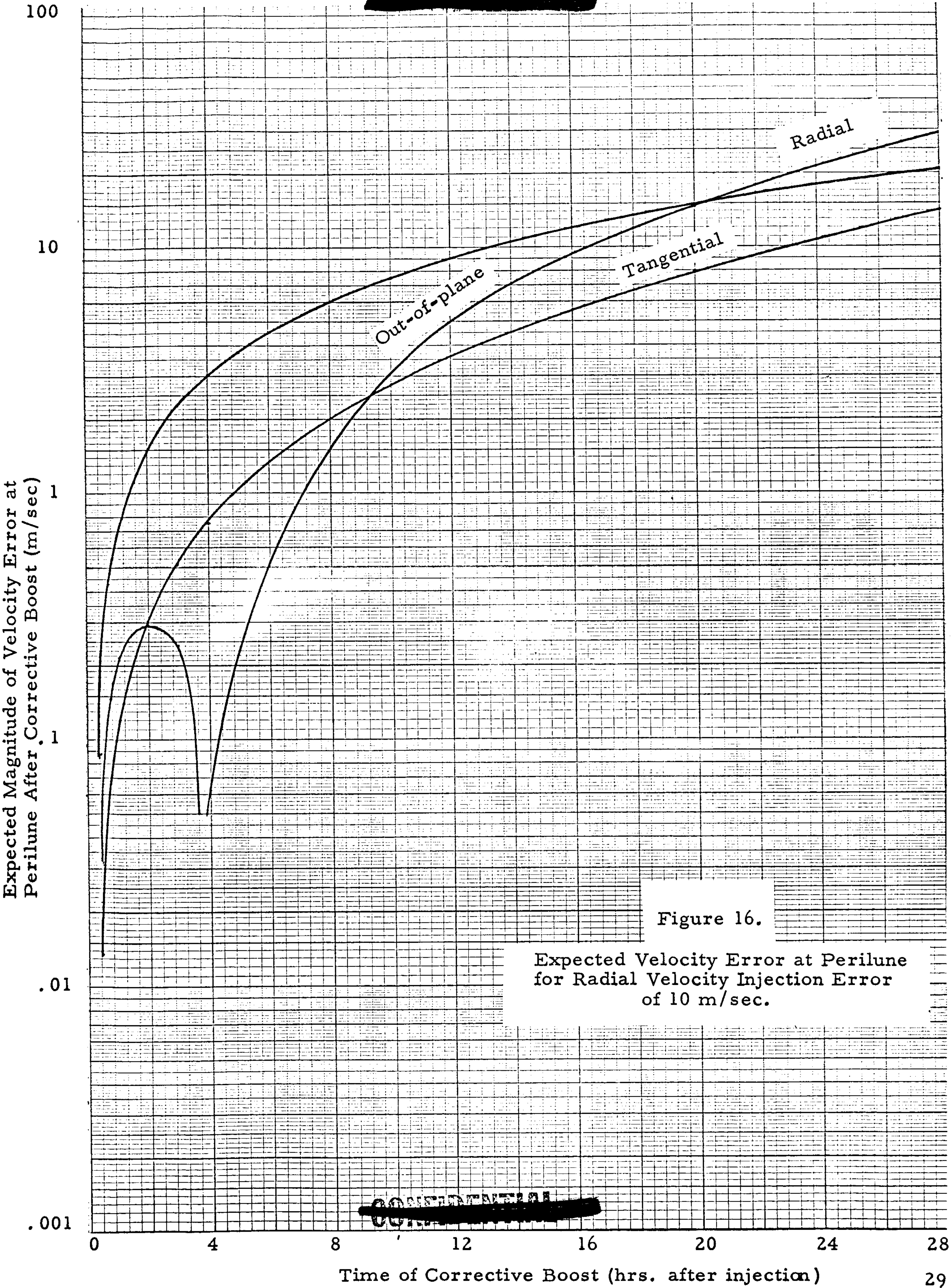

Figure 16.

Expected Velocity Error at Perilune for Radial Velocity Injection Error of 10 m/sec.

[redacted]

in the execution of commanded boosts, two boosts can easily reduce the miss at perilune to less than a nautical mile, and the total cost of this miss reduction depends only very slightly on the standard deviation of the execution error for reasonable execution errors.

It is very worthwhile to use the attitude control rockets to complete the first corrective boost in order to greatly decrease the miss following this boost. Any subsequent corrective boosts will certainly be performed using the attitude control rockets.

In the worst possible case, that in which the injection velocity error is along the velocity vector, the corrective boost required does not exceed three times the injection velocity error if the corrective boost is performed one hour after injection.

[redacted]

The Bissett-Berman Corporation 2941 Nebraska Avenue, Santa Monica, California EXbrook 4-3270

APOLLO NOTE NO. C-2
(Task 3, Item III)

H. Engel
7 October 1964

OPTIMUM CORRECTIVE BOOST PROGRAM, III

Computations have been performed for the AMPTF reference transearth trajectory to determine the required corrective boosts and the resultant errors in vacuum perigee, and in re-entry angle. The results are presented graphically for an expected injection error and also for systematically varied injection error components. Conclusions are drawn.

Transearth Reference Trajectory

The optimum midcourse correction study is to be performed for an AMPTF reference mission. The transearth injection and vacuum perigee for this mission have been specified by MSC for a patched conic approximation as

	X (n. mi.)	Y (n. mi.)	Z (n. mi.)	
Transearth Cutoff	952.74342	-332.99696	-166.43166	
Vacuum Perigee	-1900.894	2452.0558	1382.4914	
	$\dot{X}$ (ft./sec.)	$\dot{Y}$ (ft./sec.)	$\dot{Z}$ (ft./sec.)	TIME GMT
Transearth Cutoff	-2725.8340	-6776.7037	-3294.6829	265 days 1.98674 hrs.
Vacuum Perigee	-29861.431	-14211.676	-14930.665	268 days 19.13306 hrs.

for the year 1969, and using basic body-centered initial equational coordinate systems with X axis directed toward the mean equinox of date and Z axis directed parallel to the North direction along the Earth's axes. Time is referenced to 00 hours 31 December 1968. A lunar ephemeris has been supplied by MSC.

The program described in Bissett-Berman Apollo Note No. 260 has been applied, resulting in a set of patched conics for the reference mission differing slightly from that specified above because we are employing slightly different values for the physical constants and for the radius of the LSOI. For comparison, we have

	X (n. mi.)	Y (n. mi.)	Z (n. mi.)	
Transearth Cutoff	952.743	-332.997	-166.432	
Vacuum Perigee	-1945.496	2512.977	1373.556	
	$\dot{X}$ (ft./sec.)	$\dot{Y}$ (ft./sec.)	$\dot{Z}$ (ft./sec.)	Time of Flight (hrs.)
Transearth Cutoff	-2614.30	-6718.38	-3266.70	
Vacuum Perigee	-2904.75	-14684.42	-15066.22	88.71

The flight time differs by 0.4 hours, and the vacuum perigee by 0.1 n. mi., but the perigee location differs by 53.9 n. mi. Visibility of the vehicle from various MSFN stations for the injection conditions employed are shown in Figure 1. In the work presented in this note three visible stations are always used, in the range mode, with a priori range bias uncertainties of 20m, and range measurements once each minute having standard deviations of 15m. No other a priori orbit knowledge is used. As in Apollo Note No. 260, we know that ranging cannot be performed from three MSFN stations simultaneously. Instead, the stations must be time-shared. This will result in increasing the standard deviations of uncertainty in position and velocity by a factor of about $\sqrt{3}$. We shall see, however, that this does not substantially affect the conclusions drawn concerning optimum corrective boost scheduling. Further, a subsequent Apollo Note on varying time transearth trajectories will include the case of simultaneous Doppler measurements from three MSFN stations, and it will appear that that mode of operation provides greater accuracy of position and velocity estimates, which in turn will favor even more the corrective boost scheduling conclusions drawn in this note.

Reasonable assumptions about the characteristics of the MSFN radars and of the on-board guidance system lead to injection velocity errors with 3σ values of 1.68 m/sec. in the radial and out-of-plane directions and 0.90 m/sec. in the tangential direction. The largest components of position error at transearth cutoff are radial and out-of-plane, and have 3σ values of 500m.

It should be observed that all the times on the graphs in this note are measured from the instant at which the vehicle becomes visible, which is about twenty minutes after transearth injection.

Figure 2 shows the RMS miss at the nominal time of vacuum perigee after a single corrective boost, the expected value of this boost, and the standard deviation of this boost, all as functions of the time at which the corrective boost is applied. These results are for injection errors of 1.68 m/sec. radially, 0.90 m/sec. tangentically, and 1.68 m/sec. in the anthogonal direction. Figure 3 shows the corresponding expected error in re-entry angle and the standard deviation of the re-entry angle. (Re-entry occurs at 4×10^5 ft. altitude).

The RMS value of perilune miss is a function of the error in the execution of the commanded corrective boost. In this case, contrary to the translunar case described in Apollo Note No. 260, it is not a strong function of error in the execution of the commanded boost, because the execution errors are outweighed by the errors resulting from uncertainty in the desired values of corrective boost.

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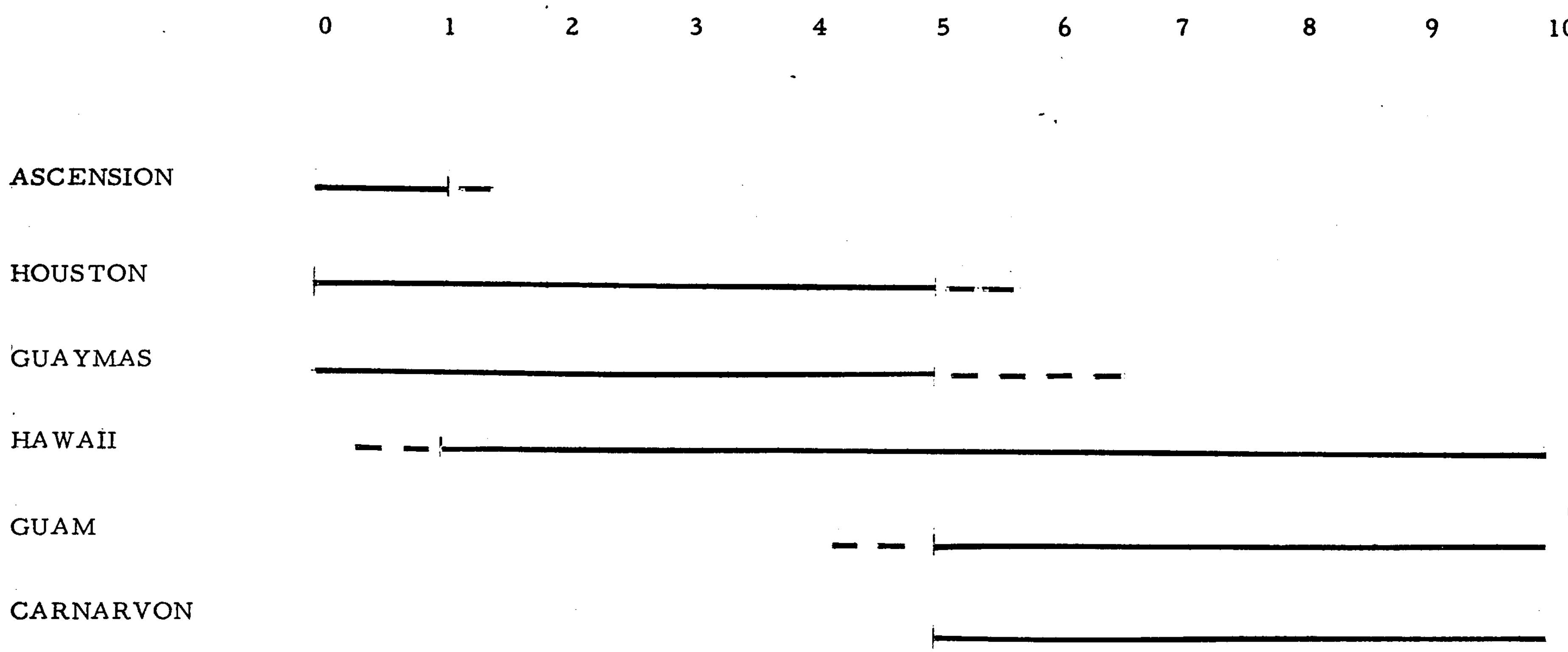

Figure 1.

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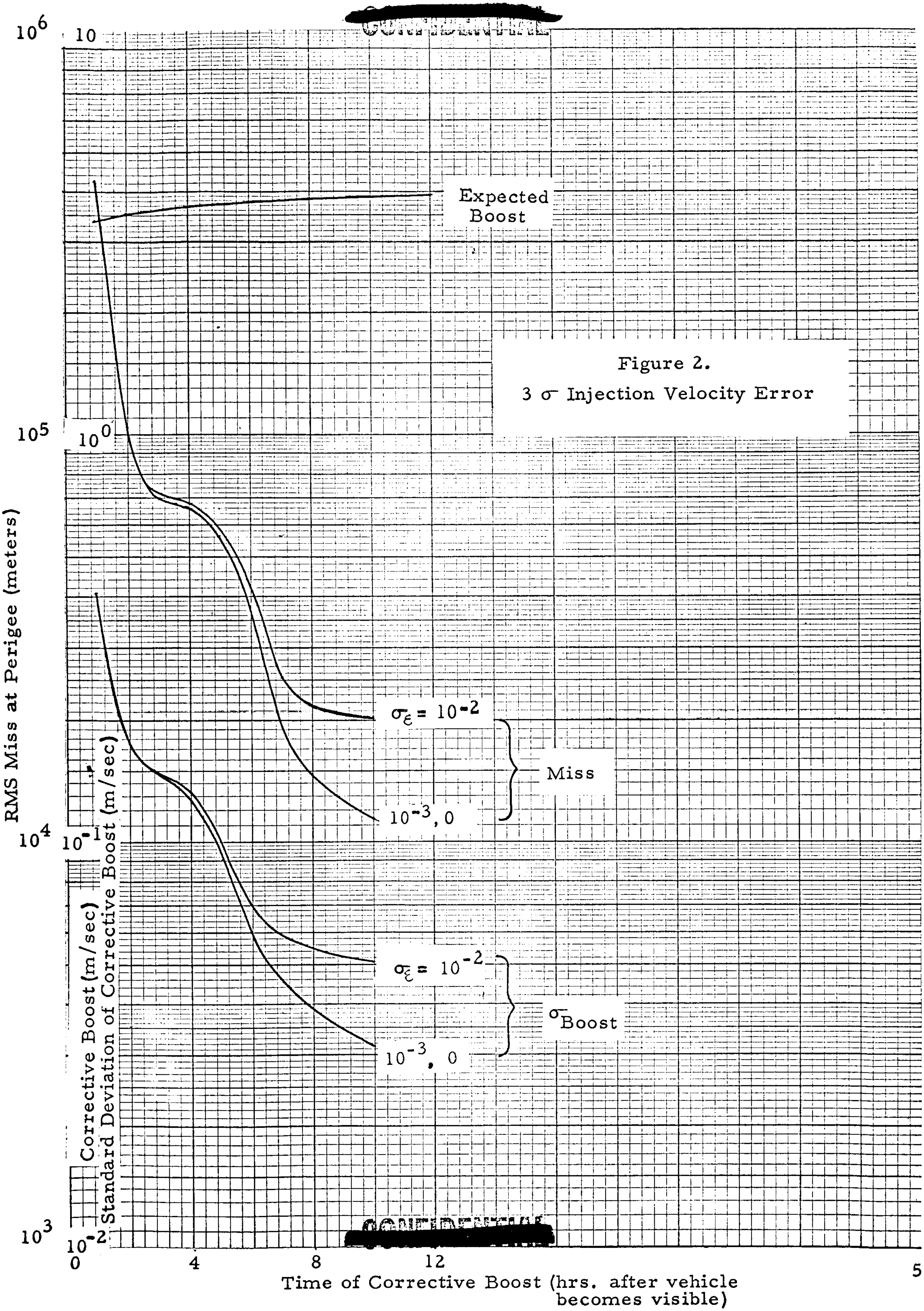

Figure 2.

3 σ Injection Velocity Error

As in the translunar corrective boost case, an arguement can be provided showing that it is best to perform the first corrective boost just one or two hours after the vehicle becomes visible, although the perilune miss might be reduced by a factor of 10 if this boost were delayed for another 8 hours. Again, the conclusion stems from the fact that even when the corrective boost is performed just two hours after the vehicle becomes visible, the remaining RMS miss after the corrective boost is about 10^5m and corresponds to the standard deviation of 0. 17 m/sec. in the corrective boost. This remaining miss could be eliminated by a boost of about 0. 17 m/sec. if a second boost could be performed immediately. This second boost cannot be performed immediately because of lack of knowledge. Even if the cost of the second boost grew at the same rate as the cost of an initial velocity error at injection, the cost of reducing this remaining error by a factor of 10 at 10 hours after the vehicle becomes visible would be just 0. 19 m/sec. (=0. 17 x 3. 92/3. 52) so the cost of these two boosts would be about 3. 52 + 0. 19m as against a cost of 3. 98m for a single corrective boost performed 10 hours after the vehicle becomes visible.

It should be noted, in Figure 3, that the standard deviation in the dive angle at re-entry far exceeds the expected value of the dive angle at re-entry, so the standard deviation rather than the expected value should be examined to determine how well the re-entry angle can be controlled after a single corrective boost. A corrective boost at two hours after the vehicle becomes visible will leave an expected dive angle error of about 0. 002 degrees with a standard deviation of about three degrees. The second, small corrective boost at ten hours (or later) will leave an expected dive angle of about 0. 002 degrees with a standard deviation of 0. 03 degrees, which should be satisfactory.

It should be pointed out that all the curves for the standard deviation of re-entry dive angle in this note have been computed using zero command execution error. However, inasmuch as the RMS miss at perilune is pretty much independent of the execution error out to six hours, the same can be expected to be true for the dive angle standard deviation. Thus, this one curve can be used for boost prior to six hours after the vehicle becomes visible for σ_{ε} equal to 0, 10^{-3} or 10^{-2}. It may also be used for the second corrective boost at any time for σ_{ε} equal to 0, 10^{-3} or 10^{-2} since the execution error in the second, small boost will be very small.

Reference Trajectory Parametric Study

The discussion of Figures 1 and 2 corresponds to a particular injection error. The necessary corrective boosts, the resultant misses at vacuum, perigee, and the re-entry dive angle errors, however, are strong functions of the injection errors. This section considers errors in position and velocity at injection parametrically. Radial, tangential and orthogonal (out-of-plane) injection position errors are considered separately. Velocity errors along the velocity vector, perpendicular to it in plane, and orthogonal (out-of-plane) are considered separately.

Figure 4 shows the RMS miss at vacuum perigee for injection position errors of 500m in the radial, tangential, and out-of-plane directions, each for corrective boost execution errors σ_{ε} of 10^{-3} and 10^{-2}. Note that for the most part the miss is independent of the execution error and depends primarily on the radar data. Figure 5 shows the magnitudes of the corresponding corrective boosts and their standard deviations.

Figure 6 shows the RMS miss at vacuum perigee for various injection velocity errors and execution errors as a function of the time of corrective boost. Figures 7, 8, and 9 show the corresponding expected magnitudes of corrective boost and the standard

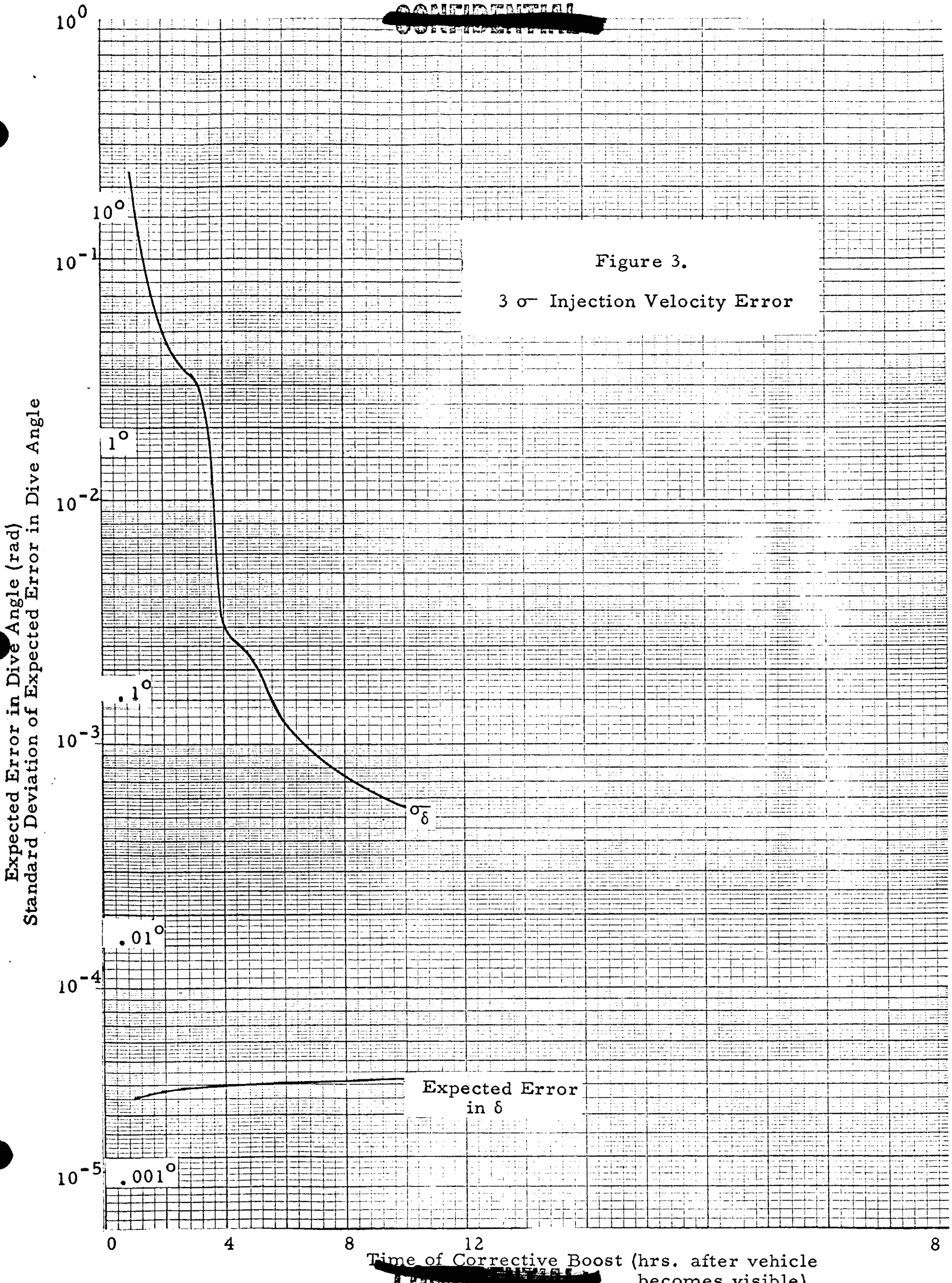

Figure 3.

3 σ Injection Velocity Error

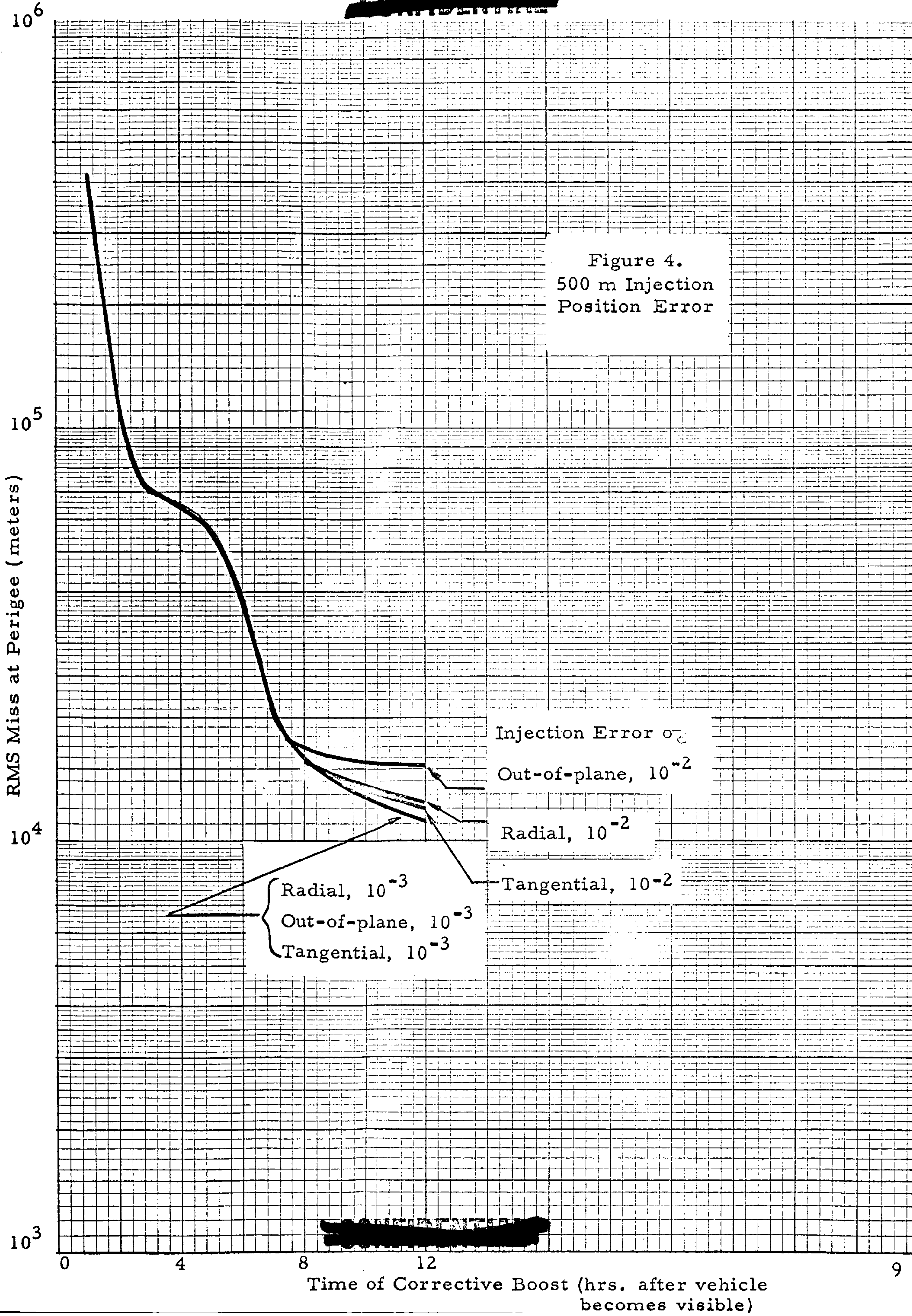

Figure 4.
500 m Injection Position Error

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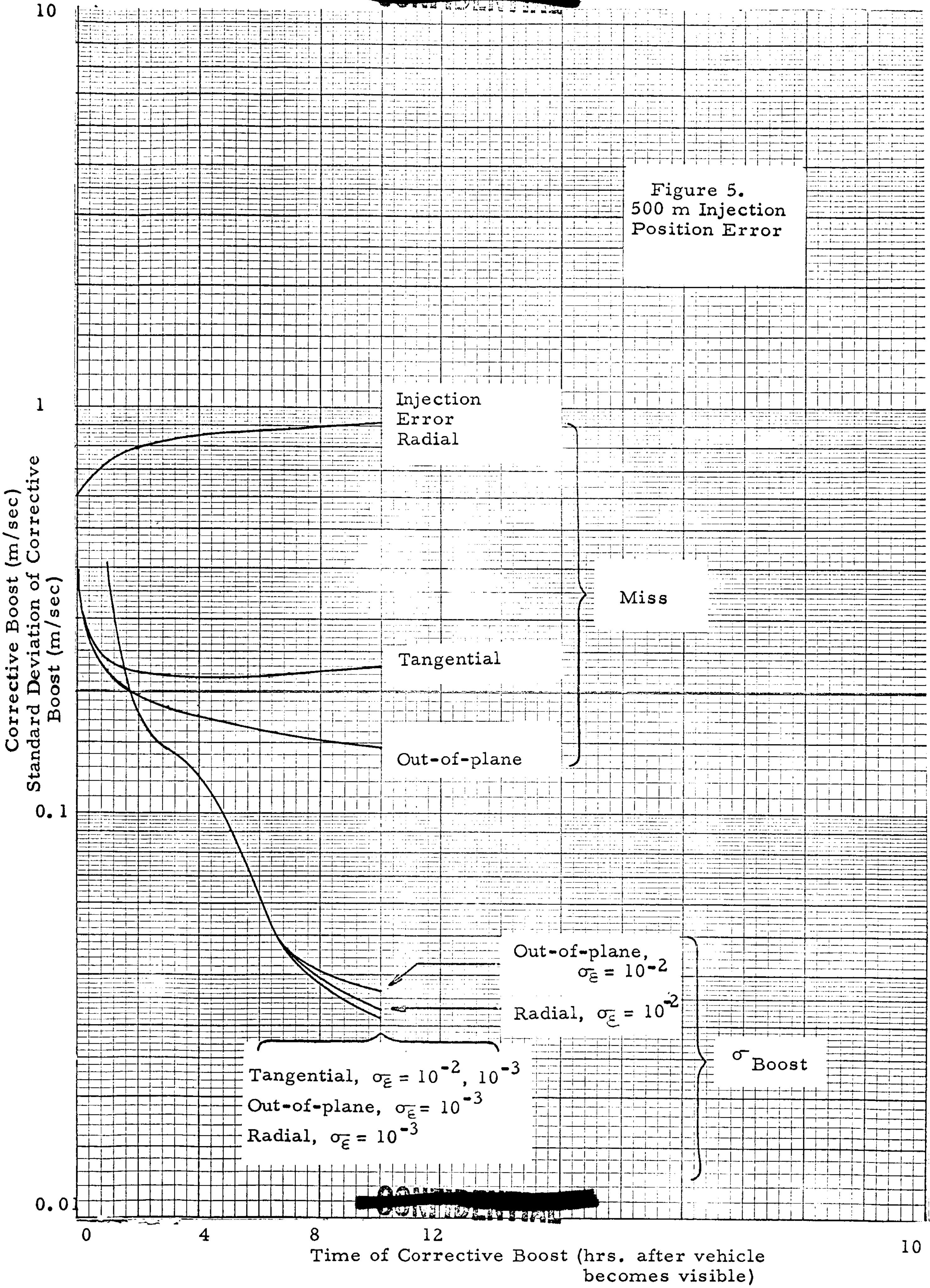

Figure 5.
500 m Injection Position Error

[redacted]

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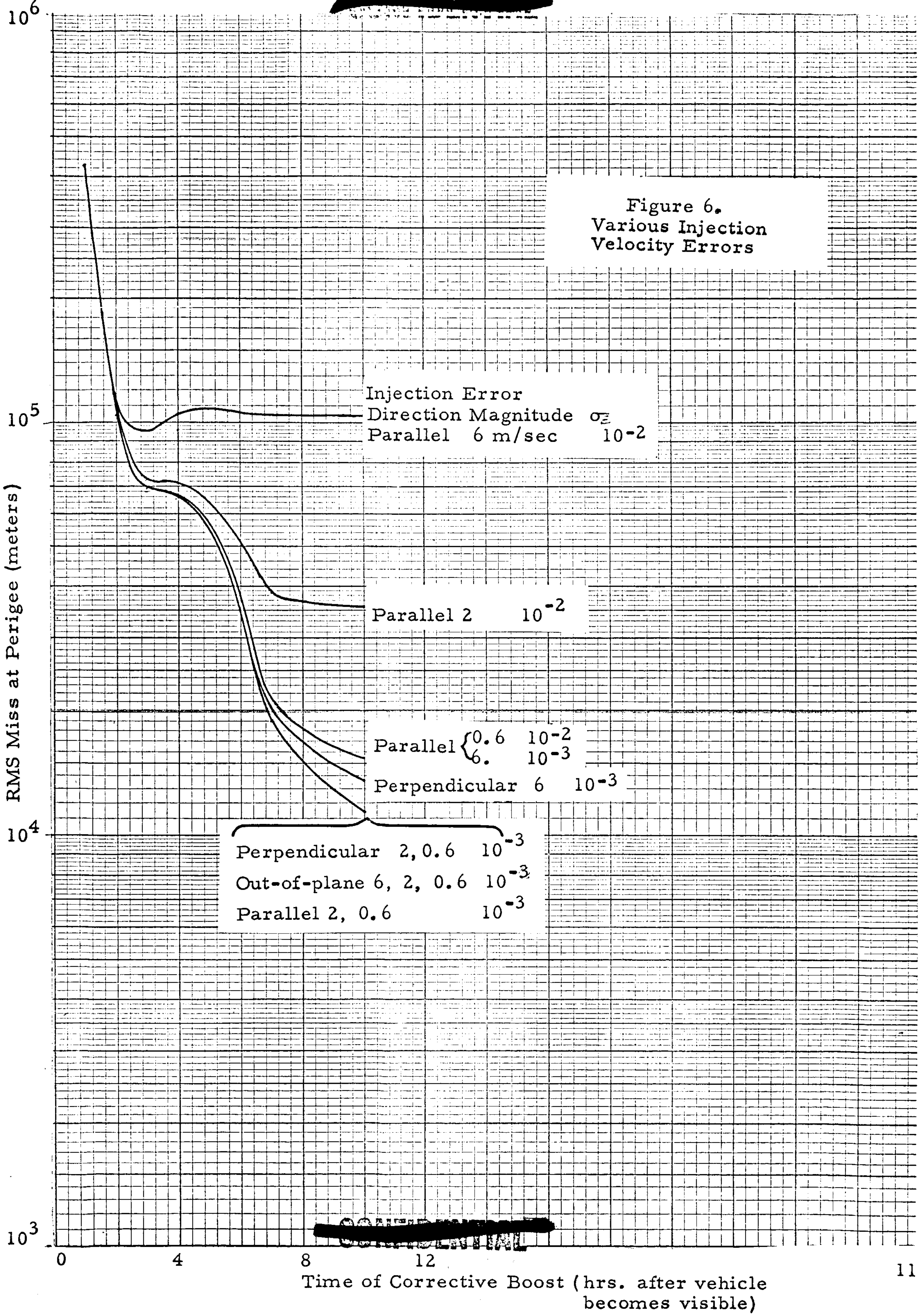

Figure 6. Various Injection Velocity Errors

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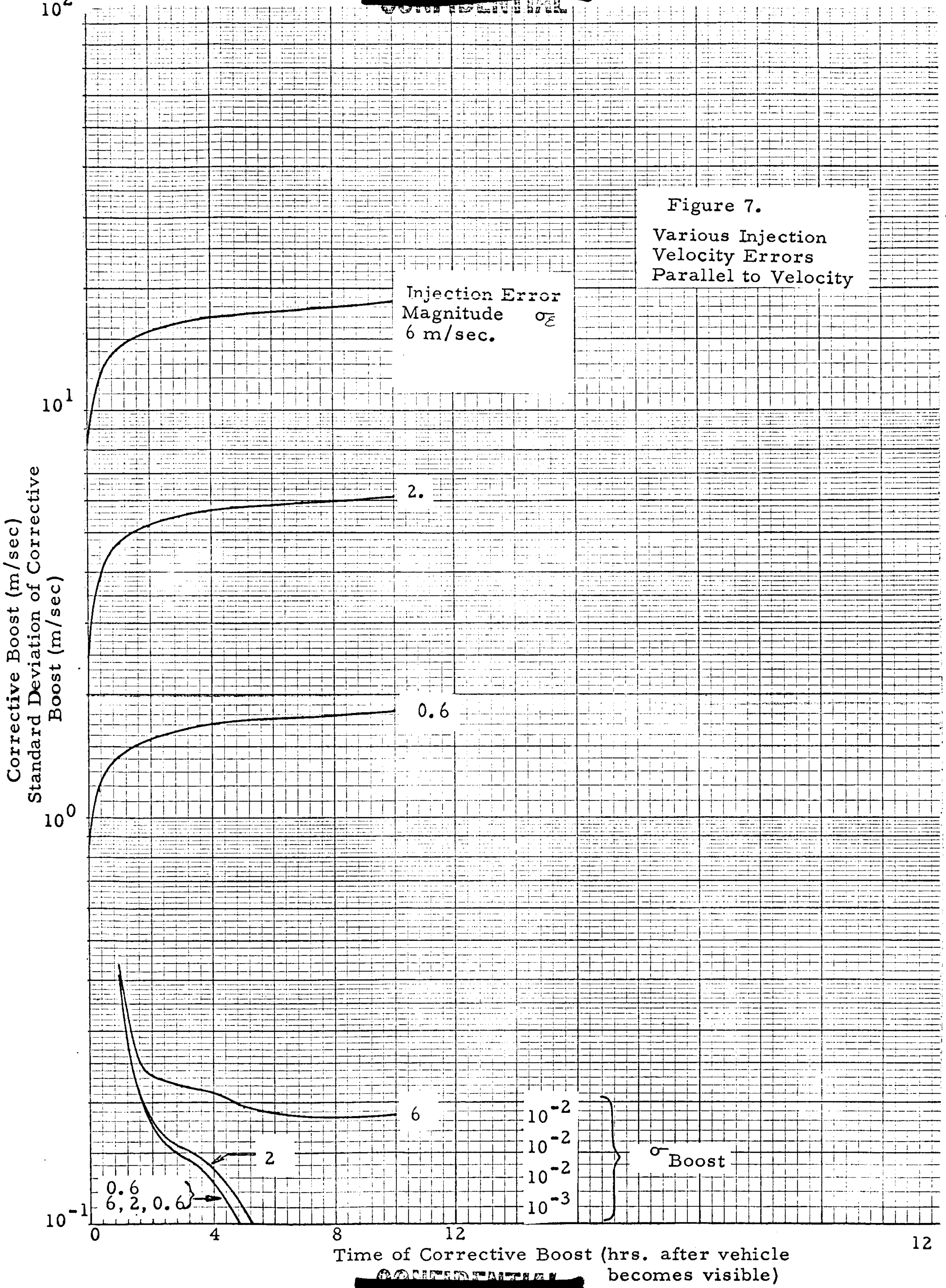

Figure 7.

Various Injection Velocity Errors Parallel to Velocity

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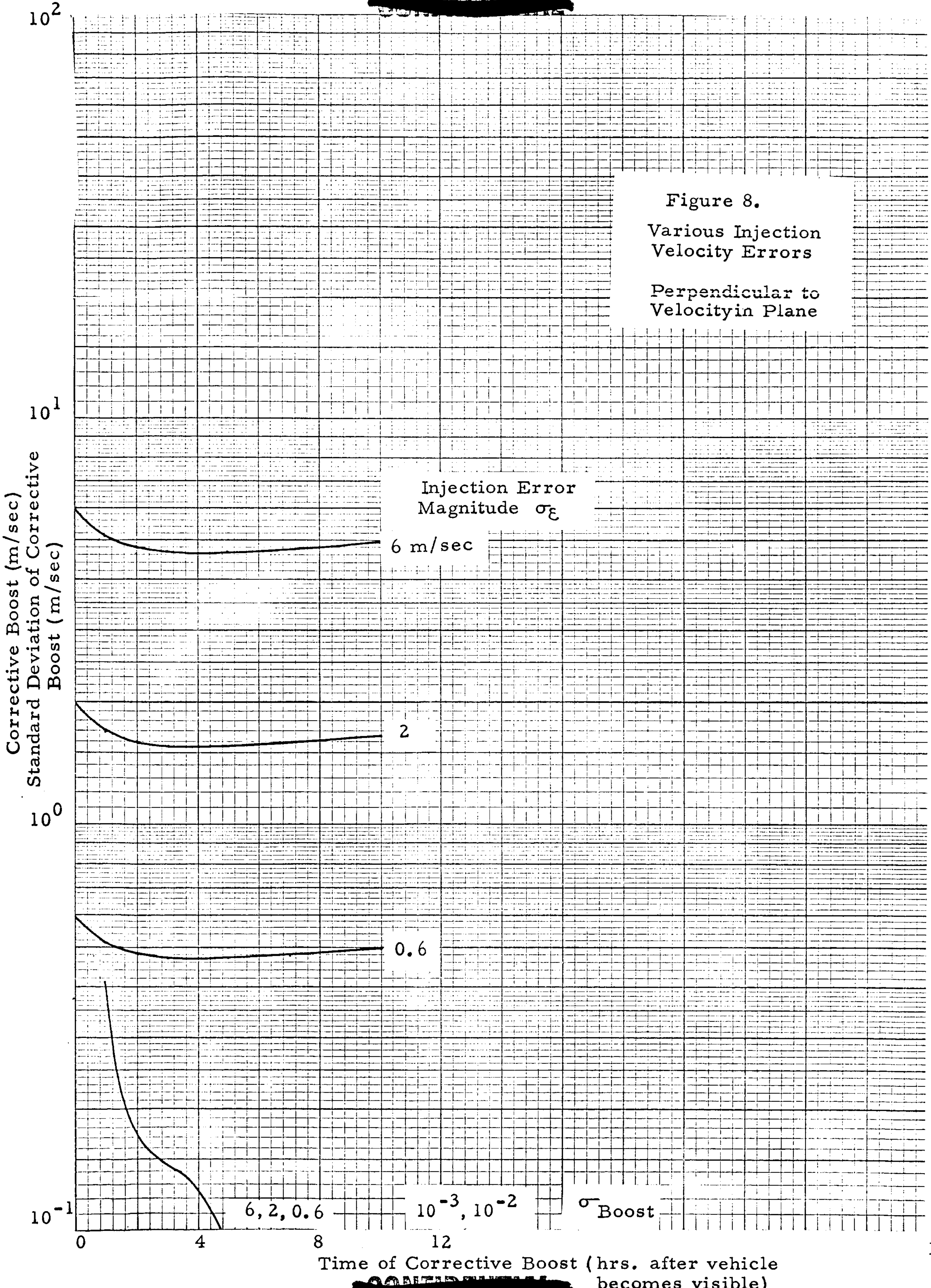

Figure 8.
Various Injection Velocity Errors

Perpendicular to Velocityin Plane

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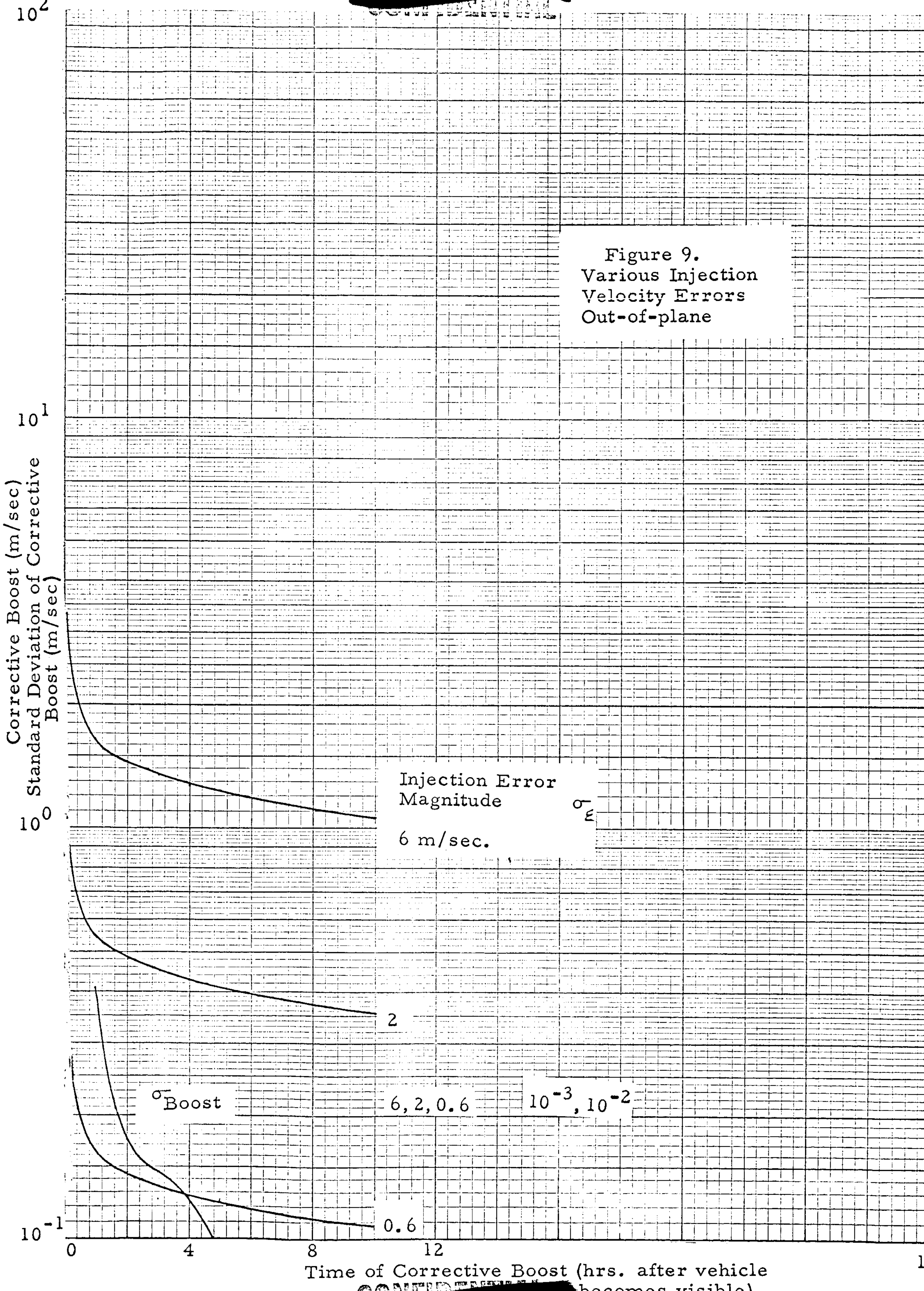

[redacted]

deviations of corrective boost.

Figure 10 shows the expected value of dive angle after the corrective boost, for various injection velocity errors. The standard deviation in the dive angle is in comparison so large as to be for the most part off this graph.

Conclusions

For a transearth flight of the order of 89 hours duration there is no difficult optimum boost scheduling problem to be solved. The first corrective boost should be made one or two hours after the vehicle becomes visible to the MSFN. A second boost will generally be required to reduce the re-entry angle error. The cost of this second boost or of any third boost is small; if the second boost is performed as late as 10 hours after the vehicle becomes visible, the cost of the second boost will still be only a few centimeters per second.

The magnitude of the first corrective boost is highly sensitive to the direction of the injection error, being largest for position errors in the radial direction and for velocity errors parallel to the velocity.

For first corrective boosts performed early, as recommended, the RMS perigee miss and the error in re-entry dive angle are primarily dependent on the ability of the MSFN to determine the vehicle position and velocity, and not upon execution errors in the commanded corrective boost. These results are based upon range measurements once per minute from each of 3 stations, with a standard deviation of 15 meters.

In the worst case, in which the injection velocity error is along the velocity vector, the corrective boost required does not exceed three times the injection velocity error if the corrective boost is performed within eight hours after injection.

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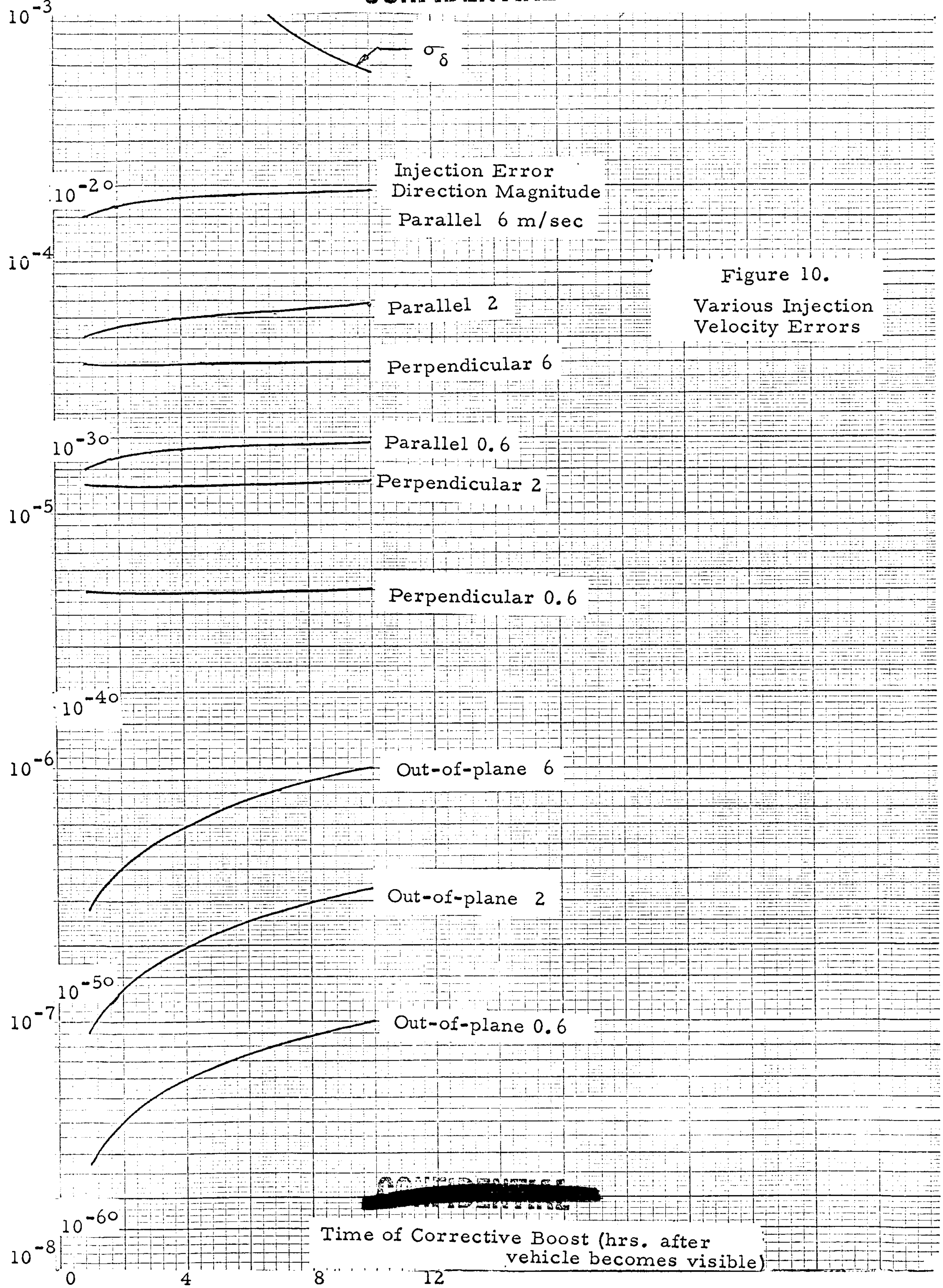

Figure 10. Various Injection Velocity Errors

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APOLLO NOTE NO. C-3
(Task 3, Item III)

H. Engel
9 October 1964

OPTIMUM CORRECTIVE BOOST PROGRAM, IV

Computations have been performed for single corrective boosts on transearth flights of varying duration, using range as the measurable. The misses at vacuum perigee, the re-entry angle errors and the corrective boost magnitudes are presented for various times of application of the corrective boost, for radial errors in injection position and for injection velocity errors along the injection velocity; these two kinds of injection errors have previously proved to require the greatest corrective boosts and to result in the greatest misses at vacuum perigee after the corrective boost.

Further, for completeness, the corrective boosts, vacuum perigee misses and re-entry angle errors are presented for transearth flight times of 70 and 110 hours for injection position errors and injection velocity errors in each of three orthogonal directions.

Still further, the magnitudes of the corrective boost, the resultant misses at vacuum perigee and the re-entry angle errors are shown for a transearth flight of 70 hours for injection position errors and injection velocity errors in each of three orthogonal directions using Doppler range rate as a measurable.

The results are presented in graphical form and conclusions drawn.

Cases

In order to obtain a family of varying time transearth flights, we have made some simplifying assumptions that make the computations easier without changing the conclusions that can be drawn from consideration of this family of flights. Two principal simplifying assumptions are that the Moon rotates about the Earth in a circular orbit in the plane of the Earth's equator, and that the flights are from perilune to perigee. These two assumptions make conic patching simpler and do not greatly change the ability of the MSFN stations to estimate space vehicle positions and velocities.

Another assumption is that the same three MSFN stations always see the vehicle. As explained in Bissett-Berman Apollo Note No. 260, this assumption results in a pessimistic estimate of the accuracy with which the MSFN can determine the position and velocity of the vehicle. This also accounts for the different shapes of the curves for RMS miss at perigee in this note and in Apollo Note No. 270. In this note the geometric aspect of the vehicle with respect to the MSFN stations changes smoothly with time and so does the perigee miss with zero execution error.

In Apollo Note No. 270, on the other hand, there are several wiggles in the graphs of perigee miss versus time, representing the changing geometric aspect of the observing stations and the vehicle, and representing changes in the observing stations.

Figure 1 shows the vacuum perigee miss for zero execution error in the performance of the corrective boost, for flight times of 70, 80, and 110 hours, with range from three stations as the measurables. The fact that the miss for a corrective boost at 16 hours after injection is not a monotonic function of the time of flight probably results from the fact that the MSFN, Moon, vehicle geometry differs with different times of flight; e. g., the transearth injection point varies with the time of flight and so does the direction of the injection velocity.

Figures 2 and 3 show the RMS miss at vacuum perigee for various flight times with an execution error σ_{ε} equal to 10^{-3}, for injection velocity errors of 2m/sec. parallel to the injection velocity and injection position errors of 500m in the radial direction, as functions of the time of the corrective boost.

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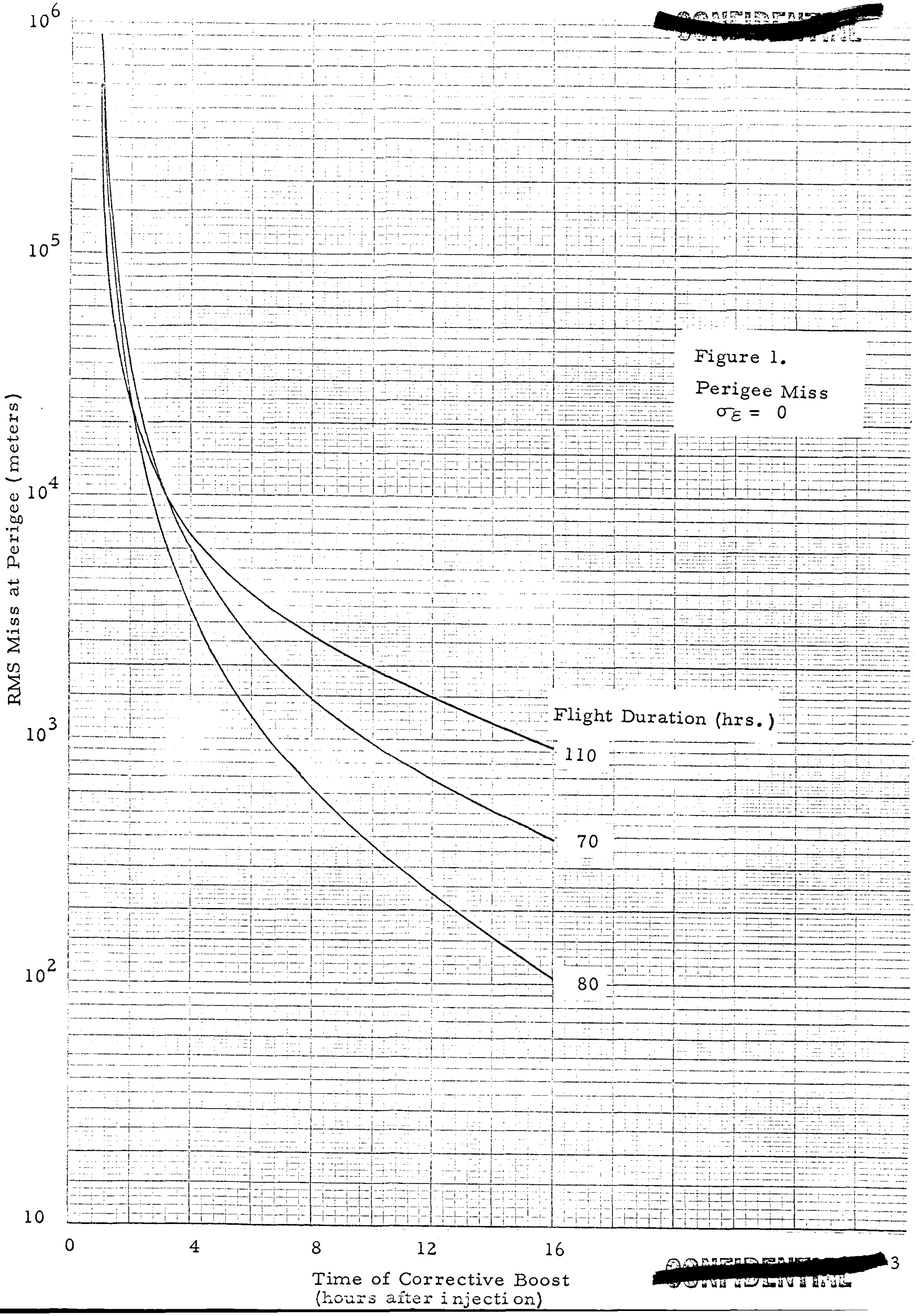

Figure 1.
Perigee Miss
$\sigma_\varepsilon = 0$

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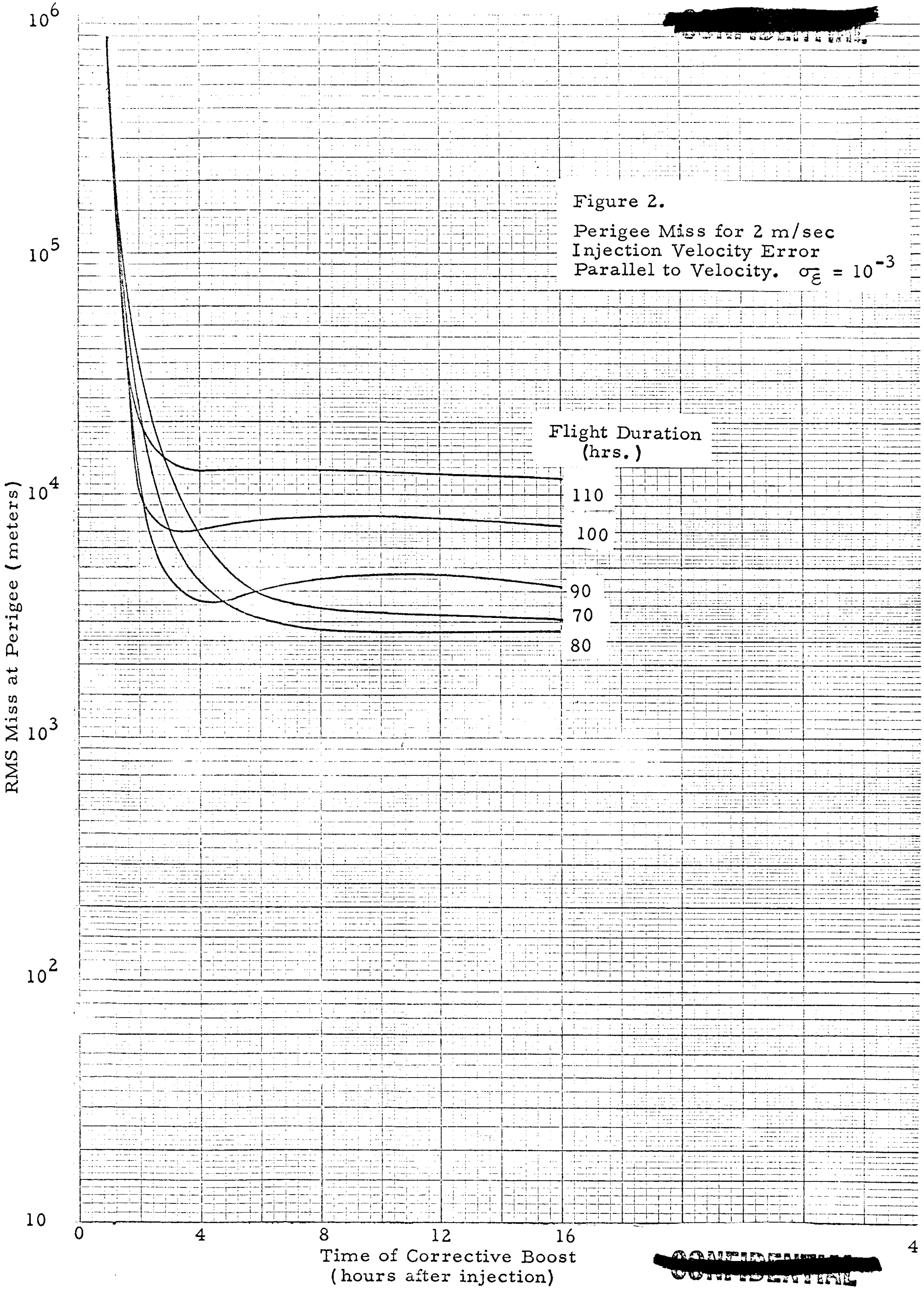

Figure 2.

Perigee Miss for 2 m/sec Injection Velocity Error Parallel to Velocity. $\sigma_{\varepsilon} = 10^{-3}$

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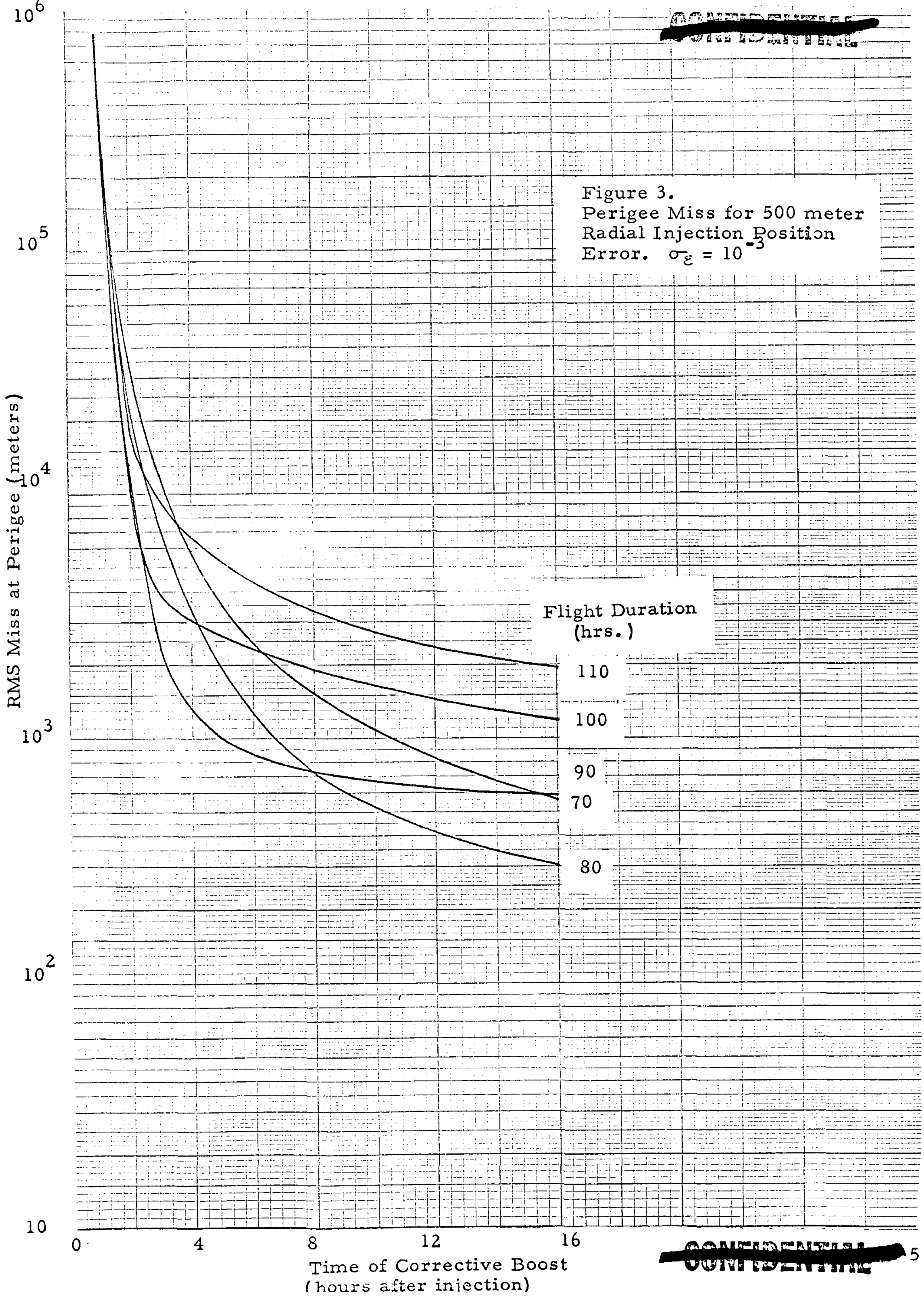

Figure 3.
Perigee Miss for 500 meter Radial Injection Position Error. $\sigma_{\varepsilon} = 10^{-3}$

[redacted]

Figures 4 and 5 show the corresponding expected errors in the re-entry dive angle, and the standard deviations of the re-entry dive angle.

Figures 6 and 7 show the expected value of corrective boost for corresponding conditions.

Figures 8 through 17 show the vacuum perigee misses, the re-entry dive angle errors, the expected corrective boosts, and the standard deviations of the corrective boosts for flight times of 70 and 110 hours with various injection position and injection velocity errors, and with various boost execution errors.

Finally, Figures 18 through 24 compare the results that may be obtained for a flight time of 70 hours if Doppler measurements from three MSFN stations are used with the results obtained if ranges from three stations are used. As before, the range measurements are taken once a minute from each station with a standard deviation of 15m and biases known a priori to 20m. The Doppler measurements are taken once a minute from each station with a standard deviation of 0.1cm/sec.

The expected value of corrective boost is independent of the MSFN accuracy and so is not shown in the comparison of range and range rate measurements. The standard deviations in the corrective boosts are so small that we have not bothered to compare them. The expected value of dive angle error is not shown because it is independent of the measurements. The standard deviation of the dive angle is shown, for zero boost execution error, since it has been calculated only for zero boost execution error.

Conclusions

For all times of flight, from 70 to 110 hours, whether the measurable is range or Dopper range-rate, the first corrective boost should be performed one or two hours after injection. This will not minimize the perigee miss or the dive angle error after the first corrective boost, but the errors remaining after this first corrective boost can be eliminated with a second corrective boost at a later time with a negligible boost cost.

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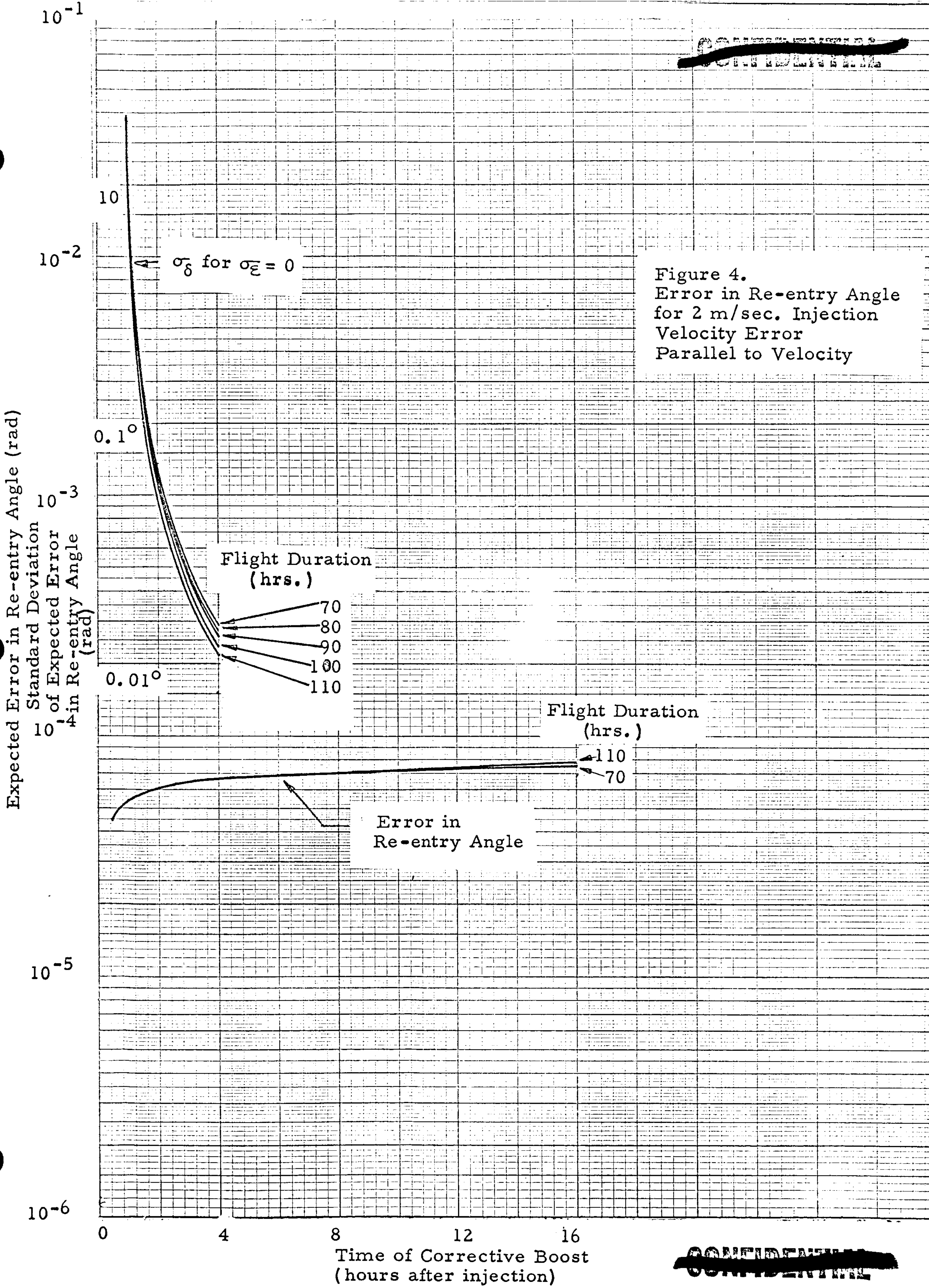

Figure 4.
Error in Re-entry Angle for 2 m/sec. Injection Velocity Error Parallel to Velocity

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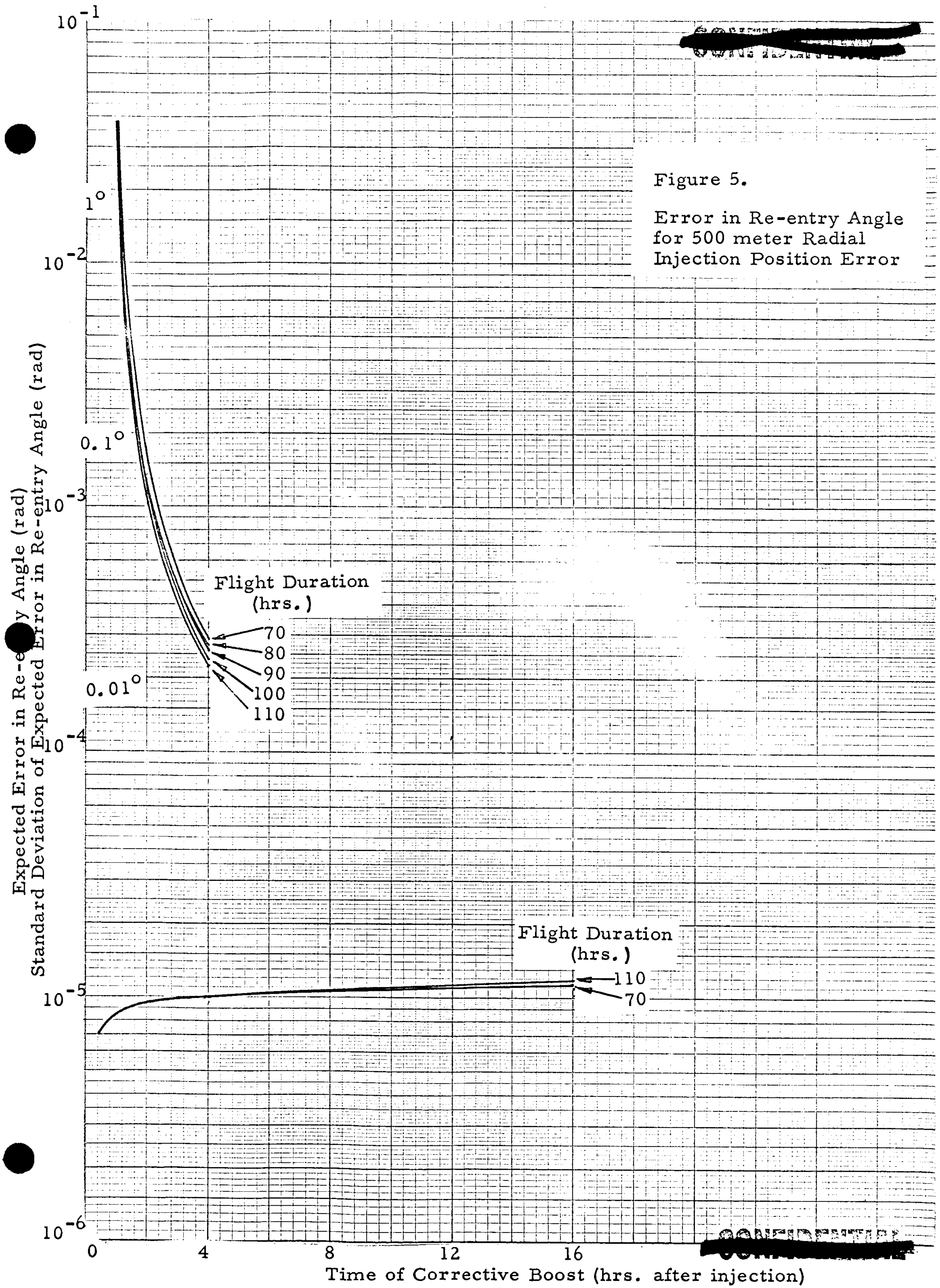

Figure 5.

Error in Re-entry Angle for 500 meter Radial Injection Position Error

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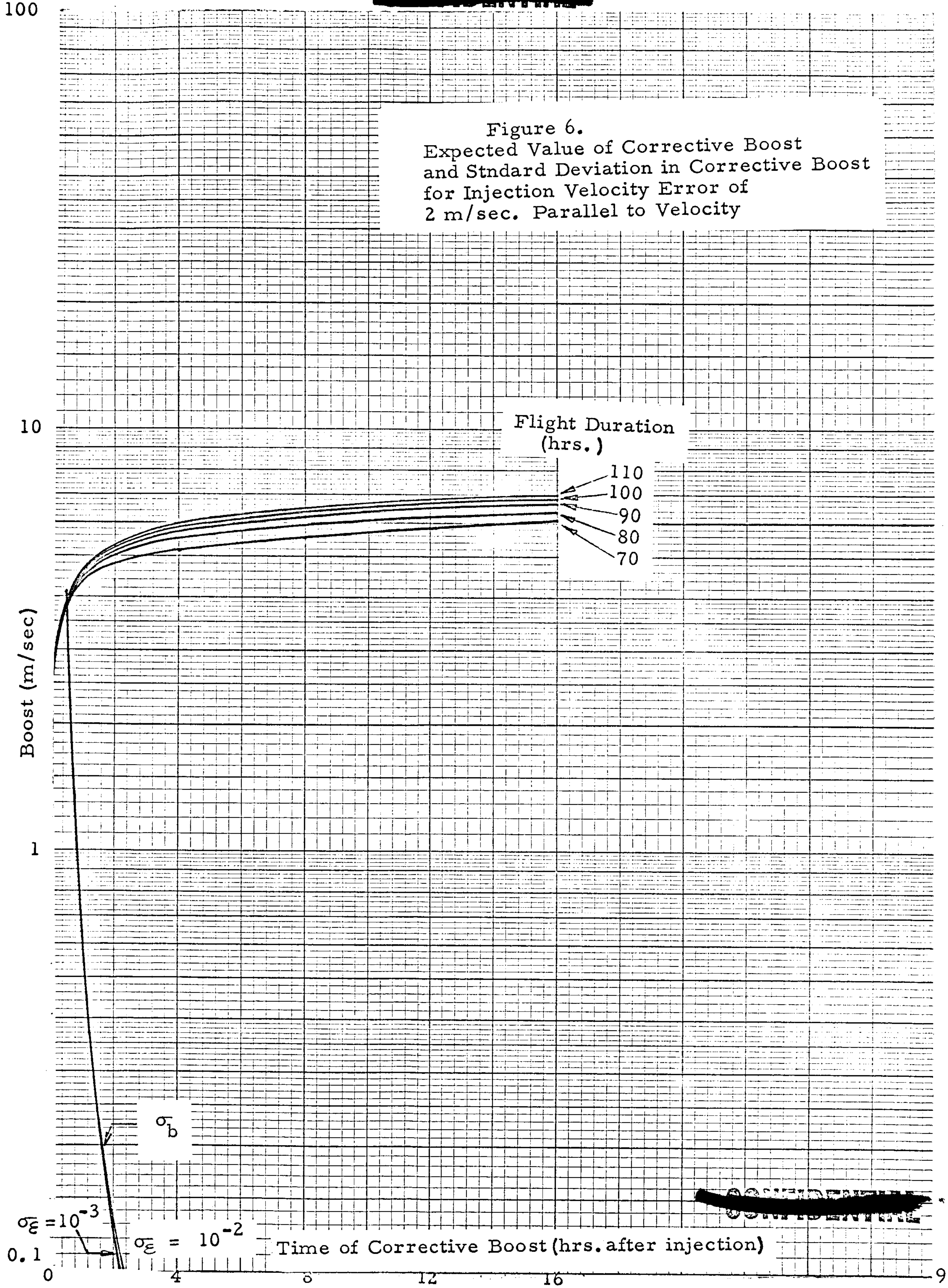

Figure 6.
Expected Value of Corrective Boost and Stndard Deviation in Corrective Boost for Injection Velocity Error of 2 m/sec. Parallel to Velocity

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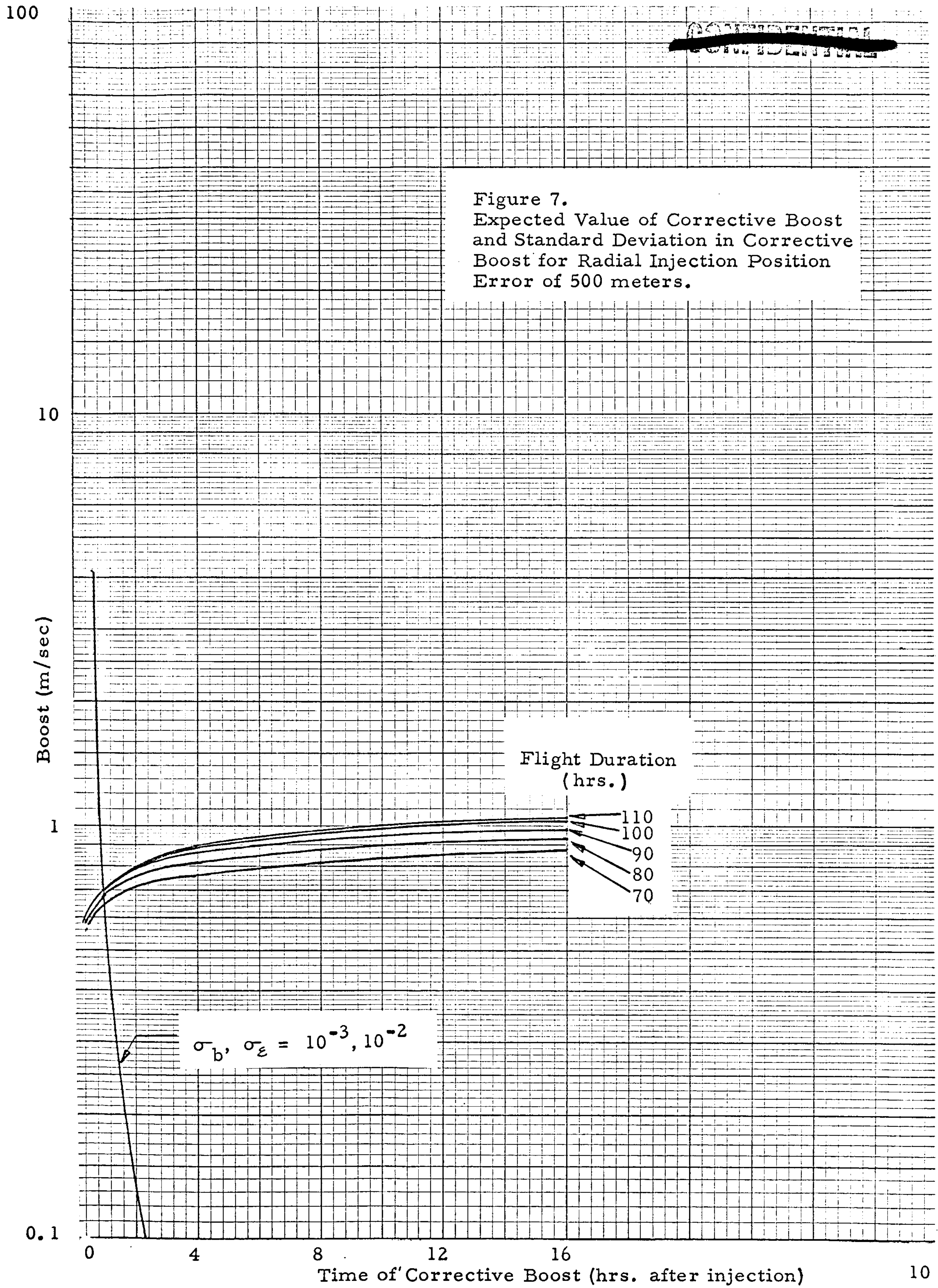

Figure 7.
Expected Value of Corrective Boost and Standard Deviation in Corrective Boost for Radial Injection Position Error of 500 meters.

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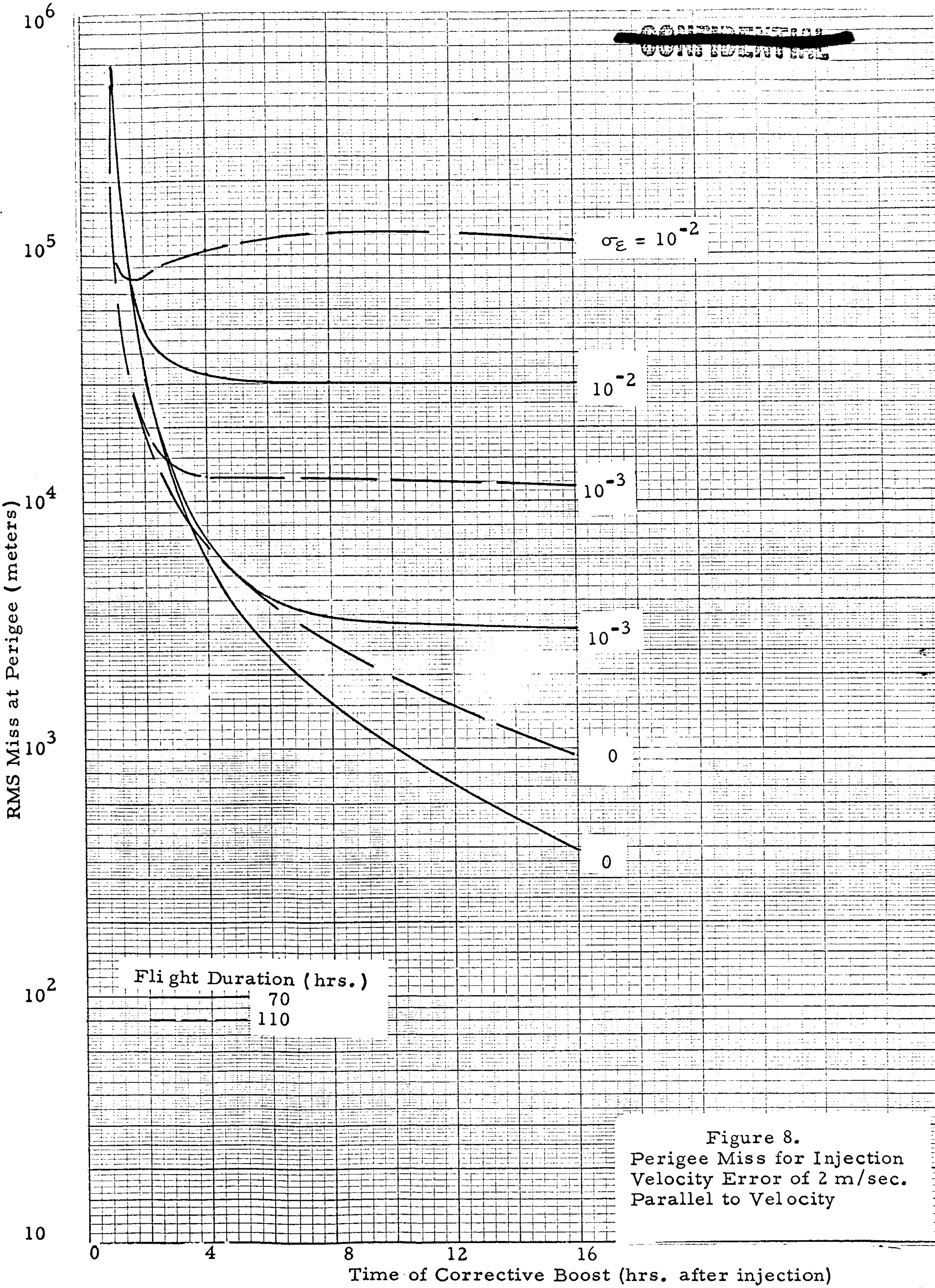

Figure 8.
Perigee Miss for Injection Velocity Error of 2 m/sec. Parallel to Velocity

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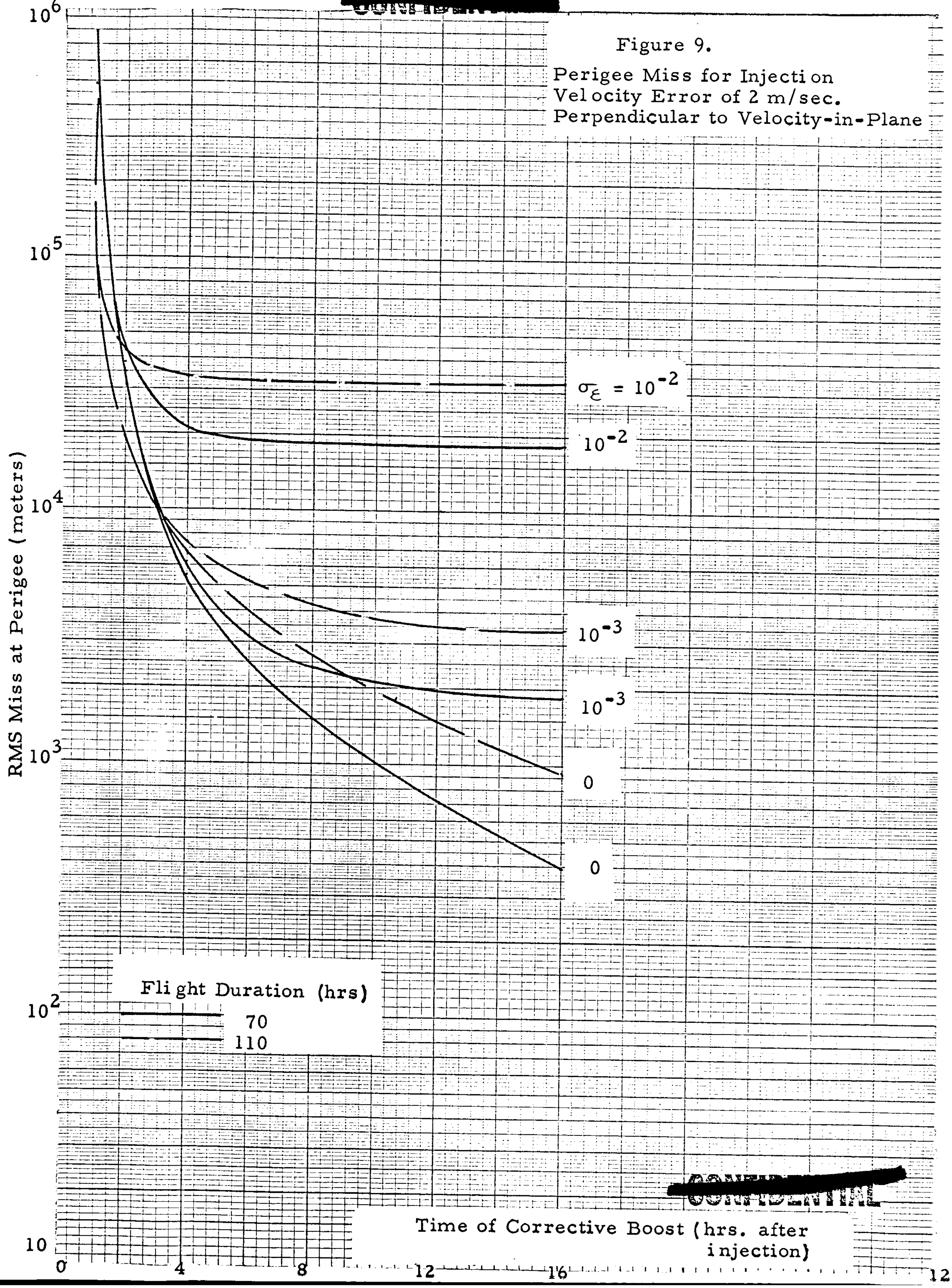

Figure 9.
Perigee Miss for Injection Velocity Error of 2 m/sec. Perpendicular to Velocity-in-Plane

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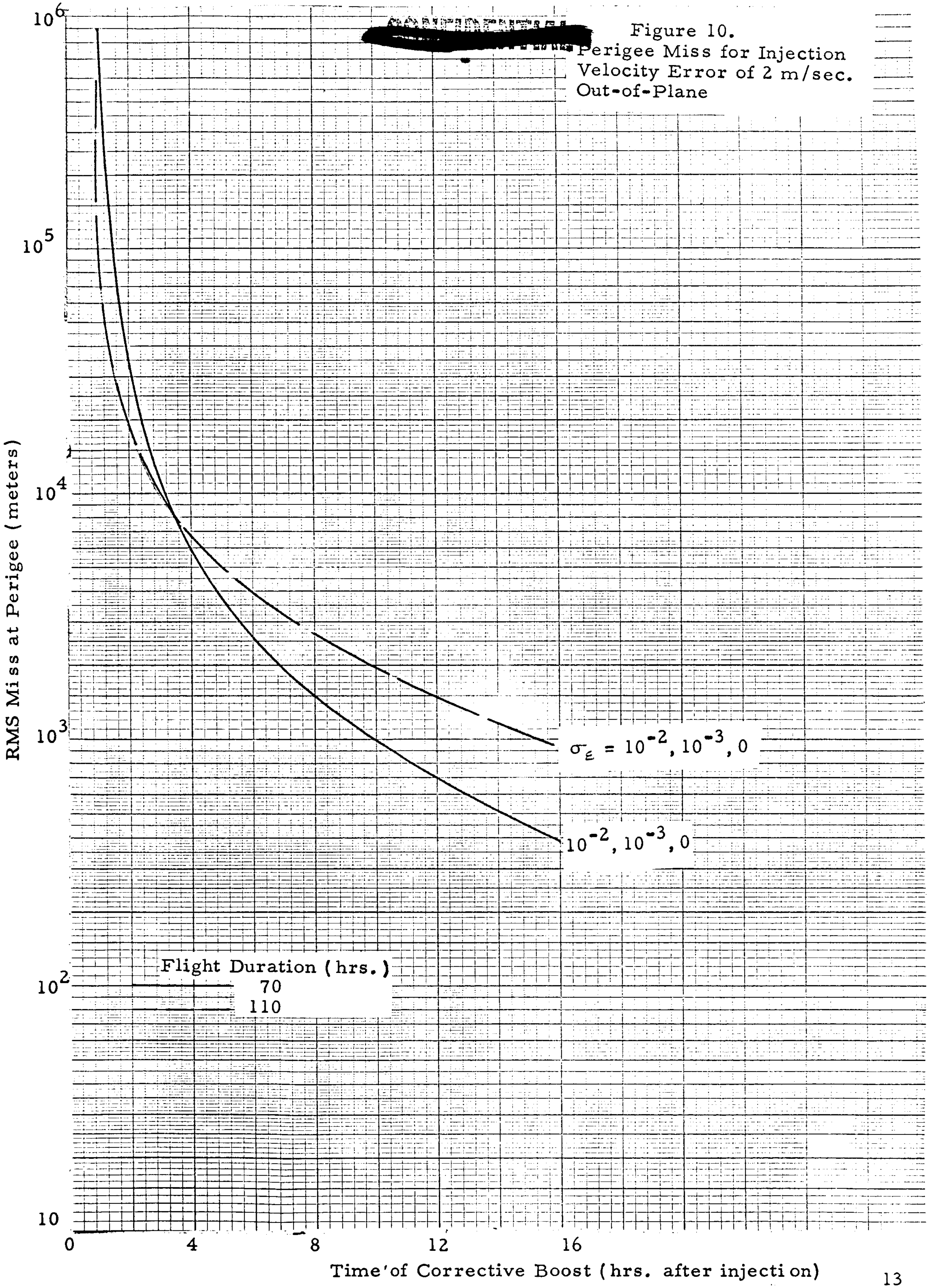

Figure 10. Perigee Miss for Injection Velocity Error of 2 m/sec. Out-of-Plane

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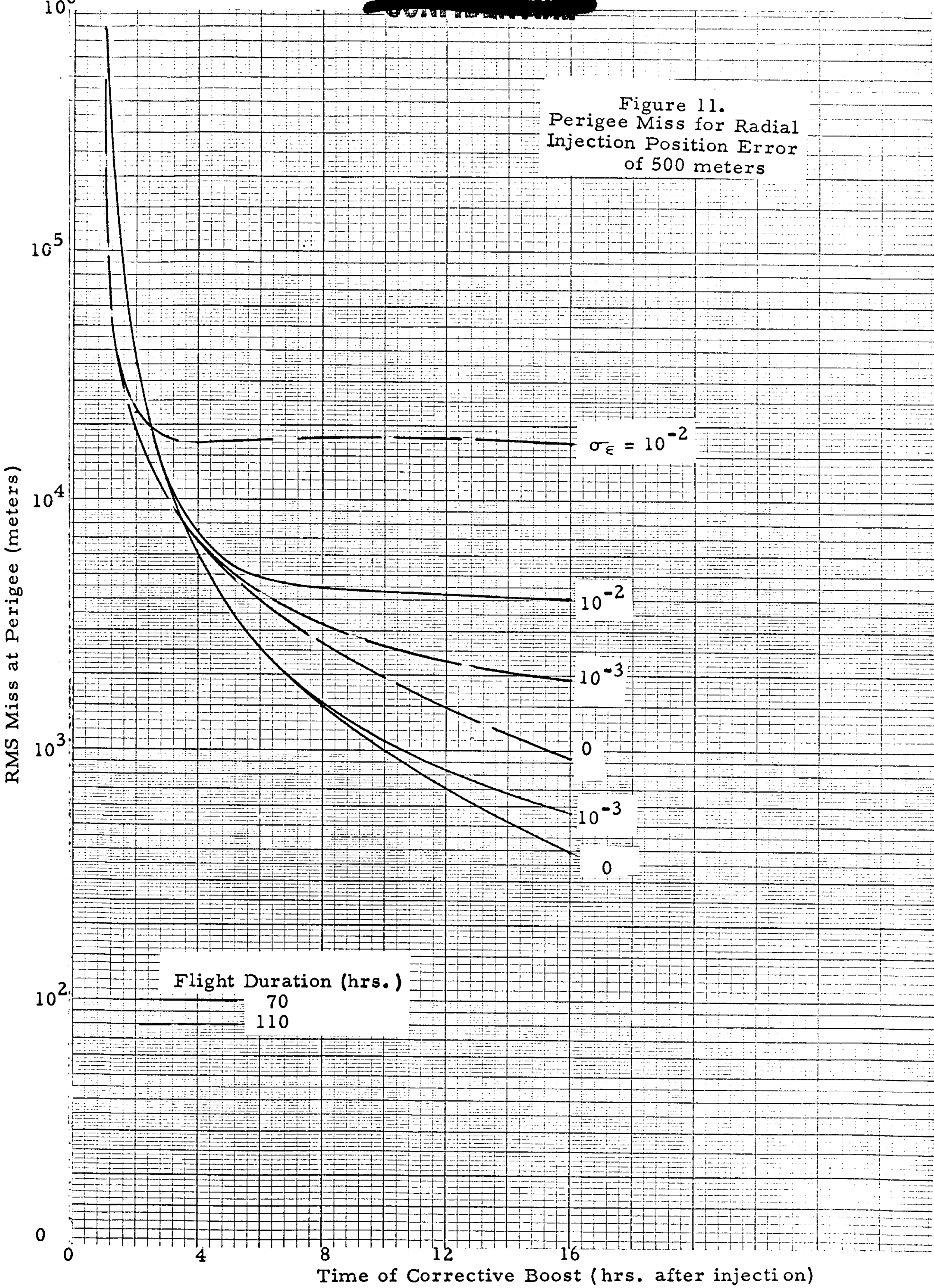

Figure 11. Perigee Miss for Radial Injection Position Error of 500 meters

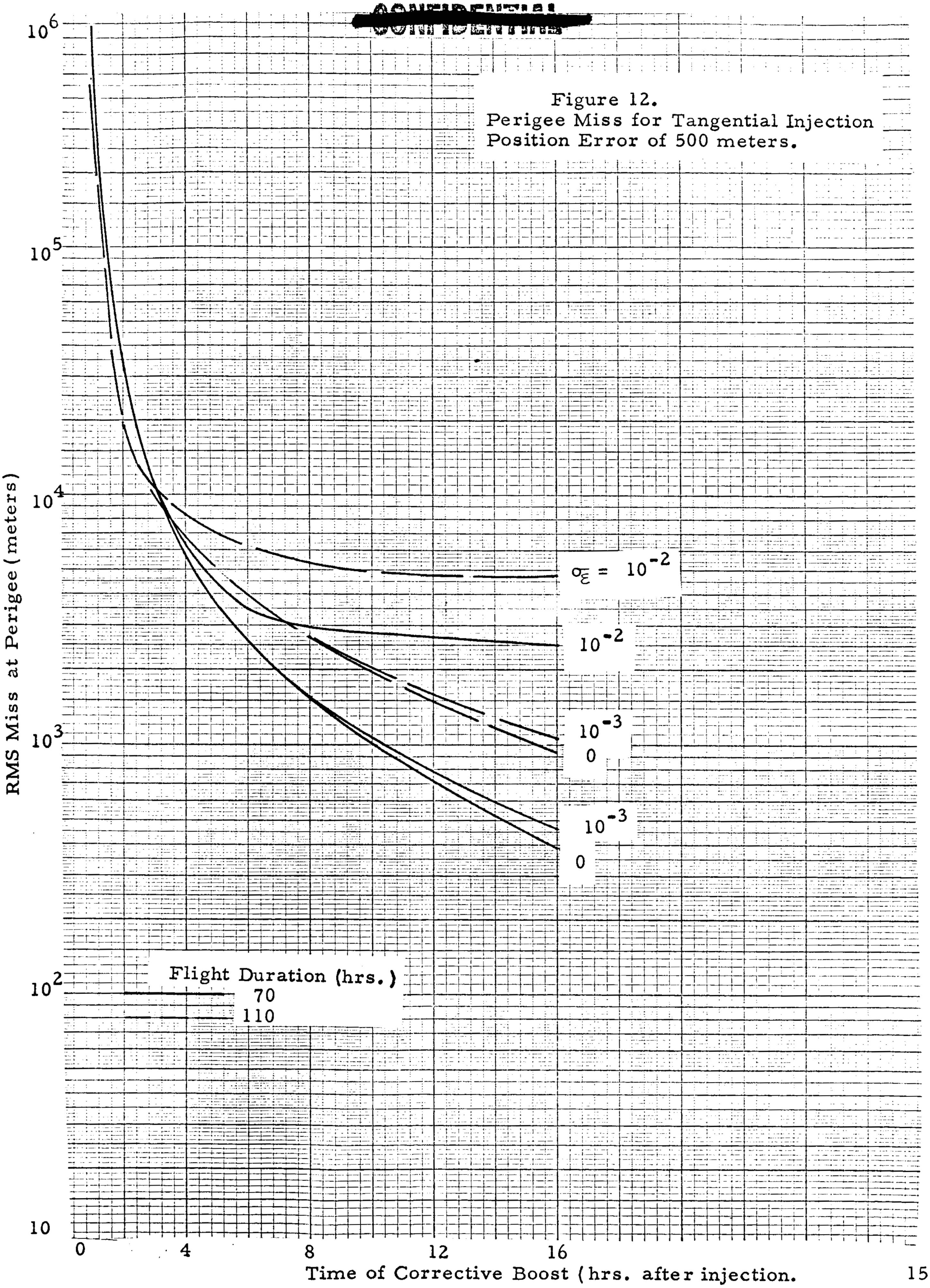

Figure 12.
Perigee Miss for Tangential Injection Position Error of 500 meters.

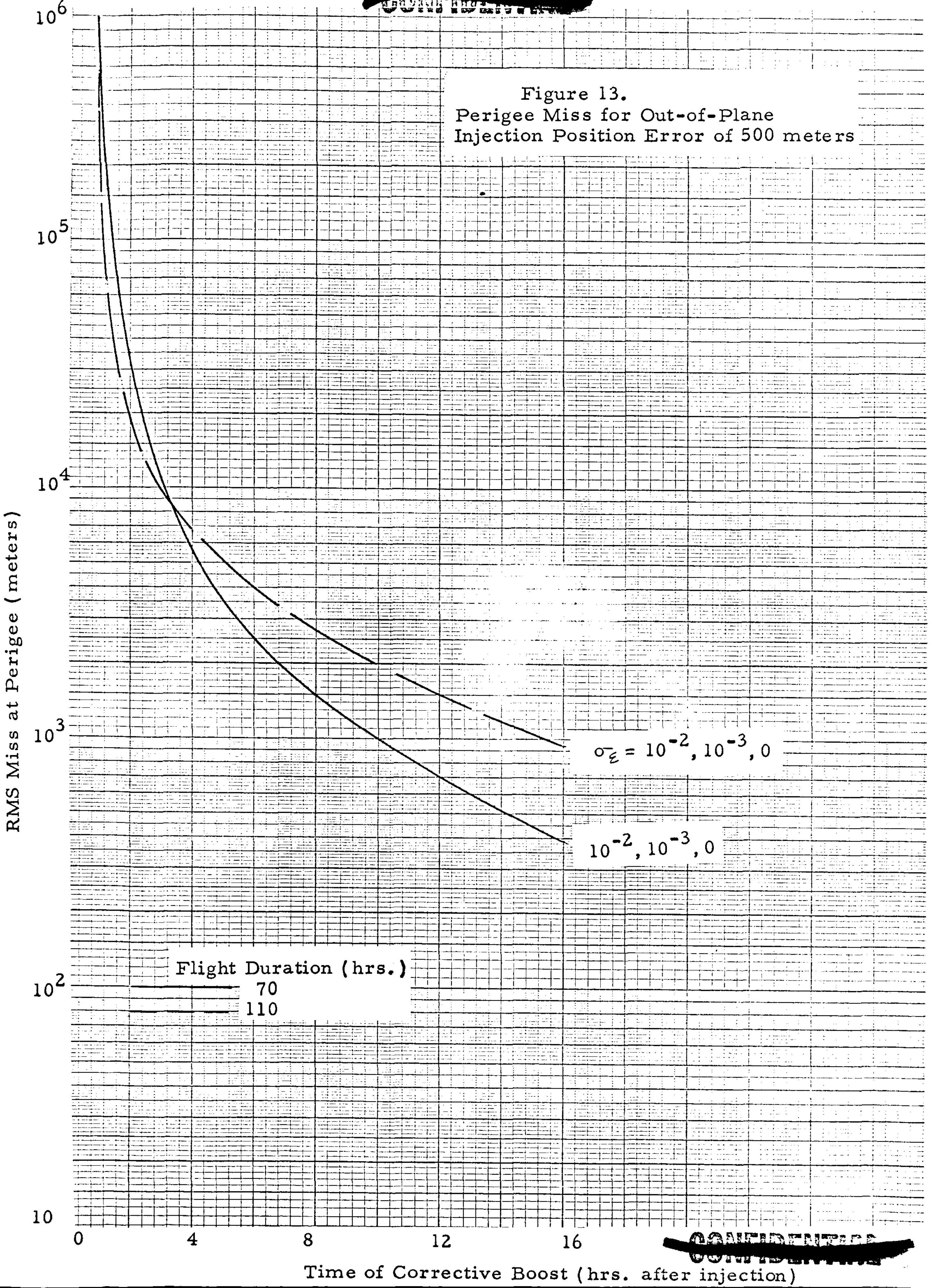

Figure 13.
Perigee Miss for Out-of-Plane
Injection Position Error of 500 meters

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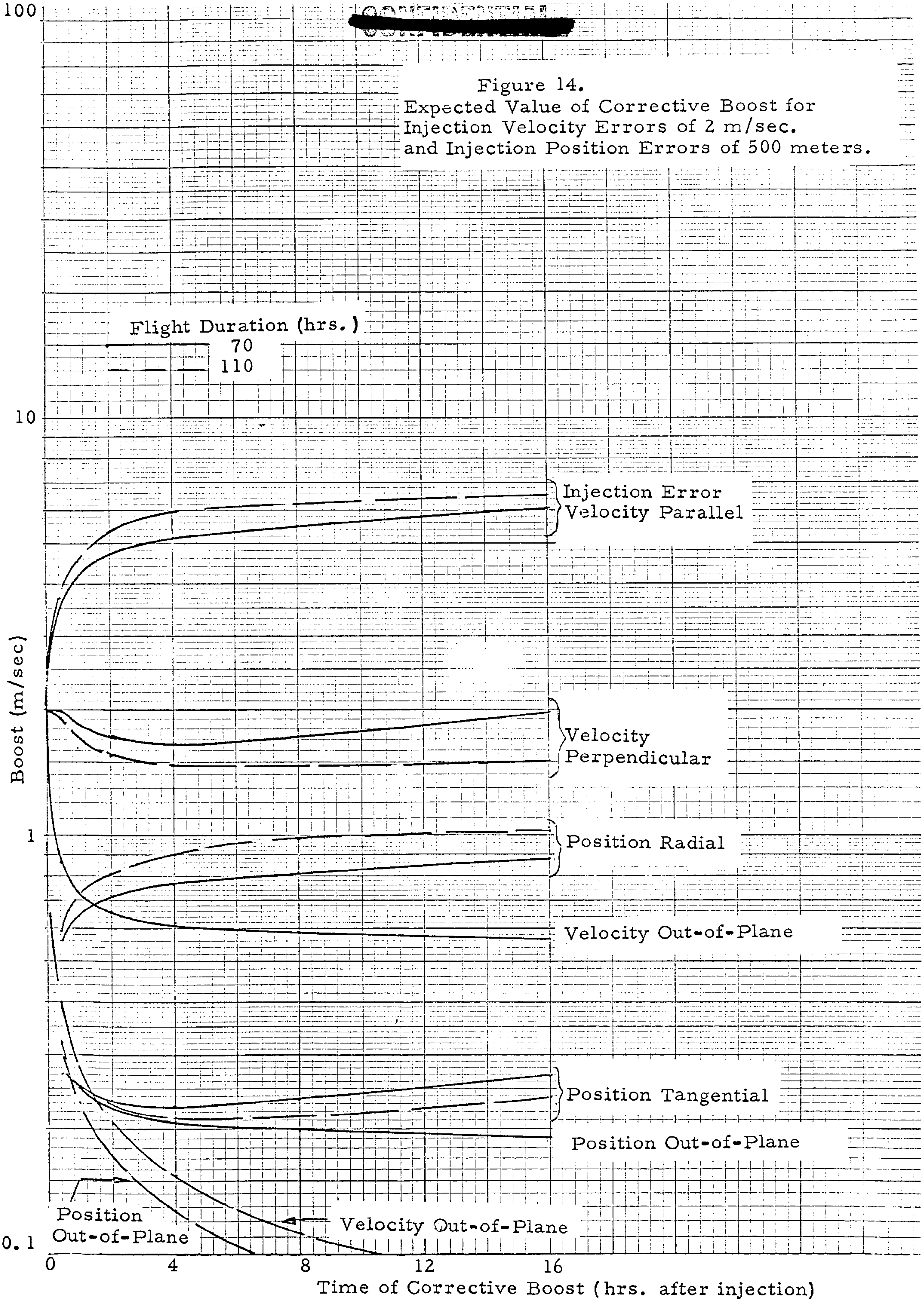

Figure 14.
Expected Value of Corrective Boost for Injection Velocity Errors of 2 m/sec. and Injection Position Errors of 500 meters.

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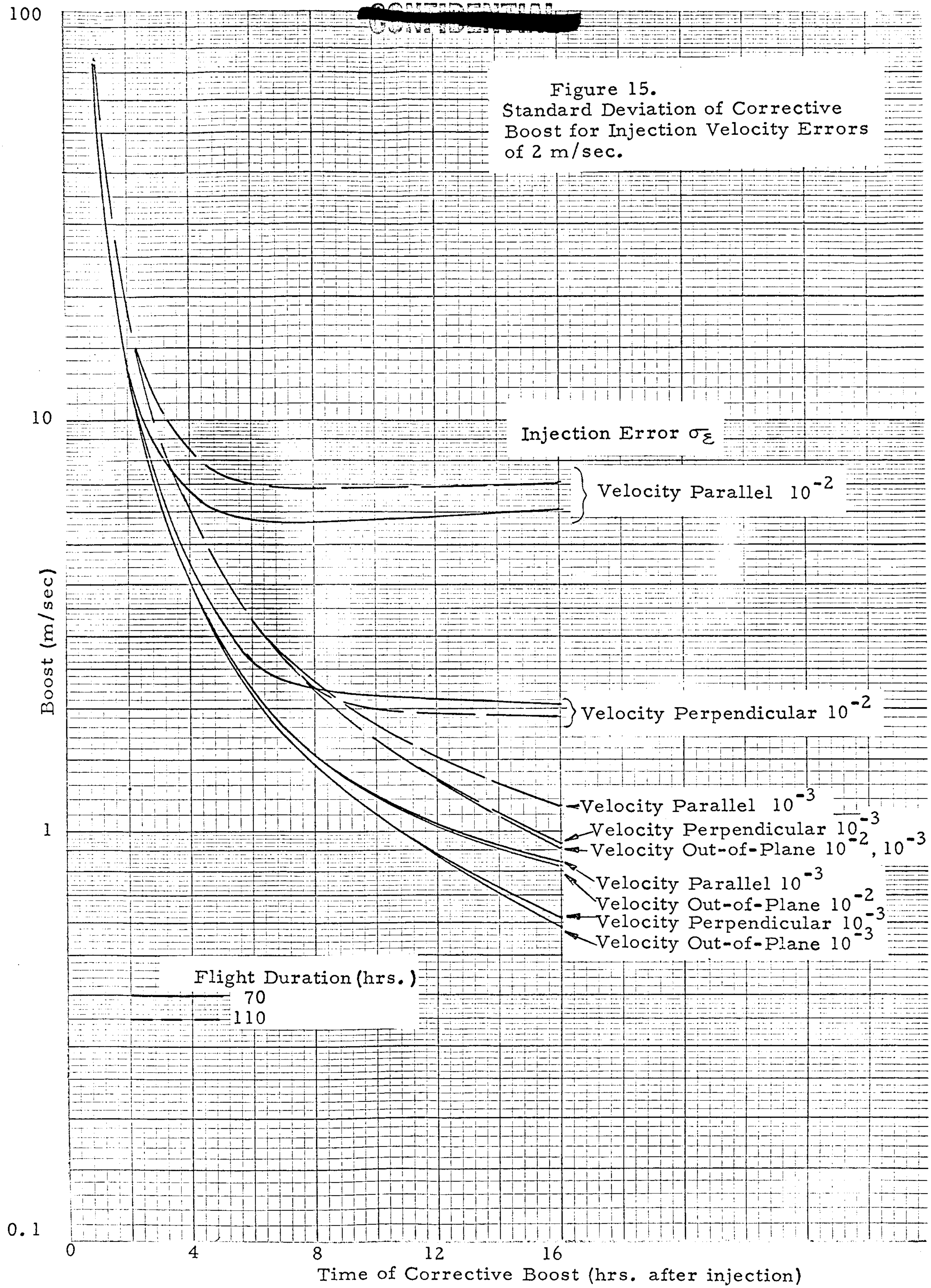

Figure 15.
Standard Deviation of Corrective Boost for Injection Velocity Errors of 2 m/sec.

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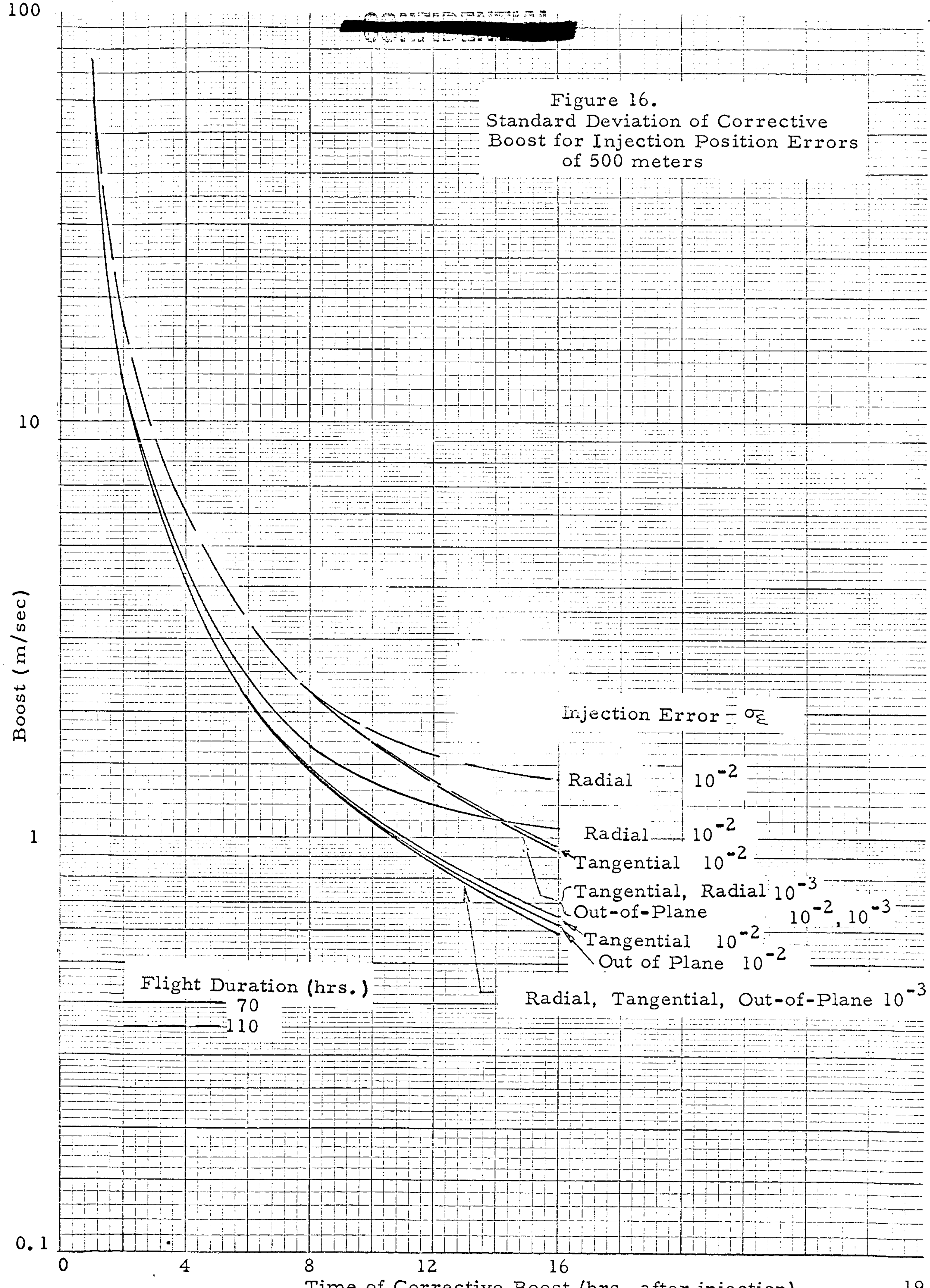

Figure 16. Standard Deviation of Corrective Boost for Injection Position Errors of 500 meters

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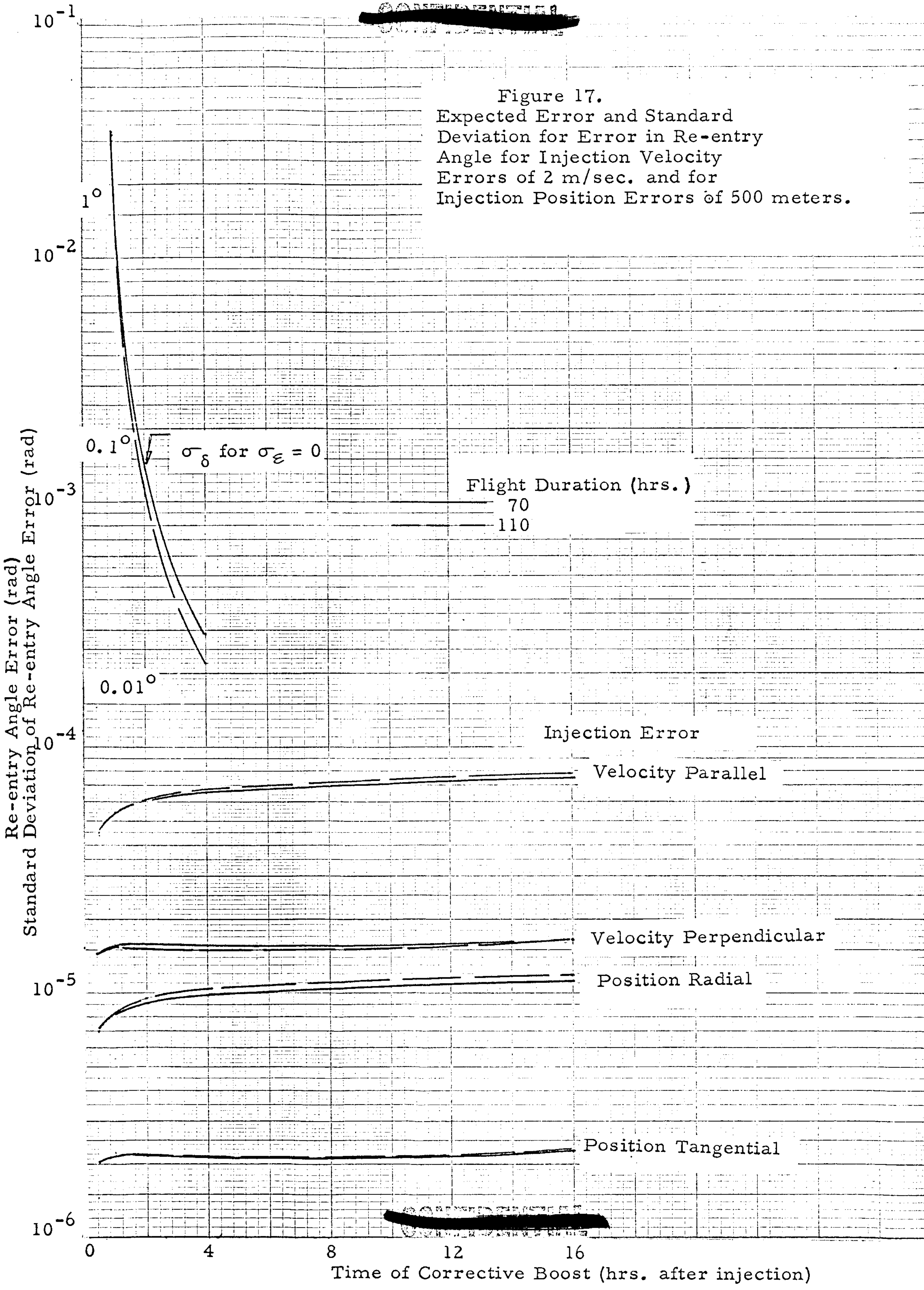

Figure 17.
Expected Error and Standard Deviation for Error in Re-entry Angle for Injection Velocity Errors of 2 m/sec. and for Injection Position Errors of 500 meters.

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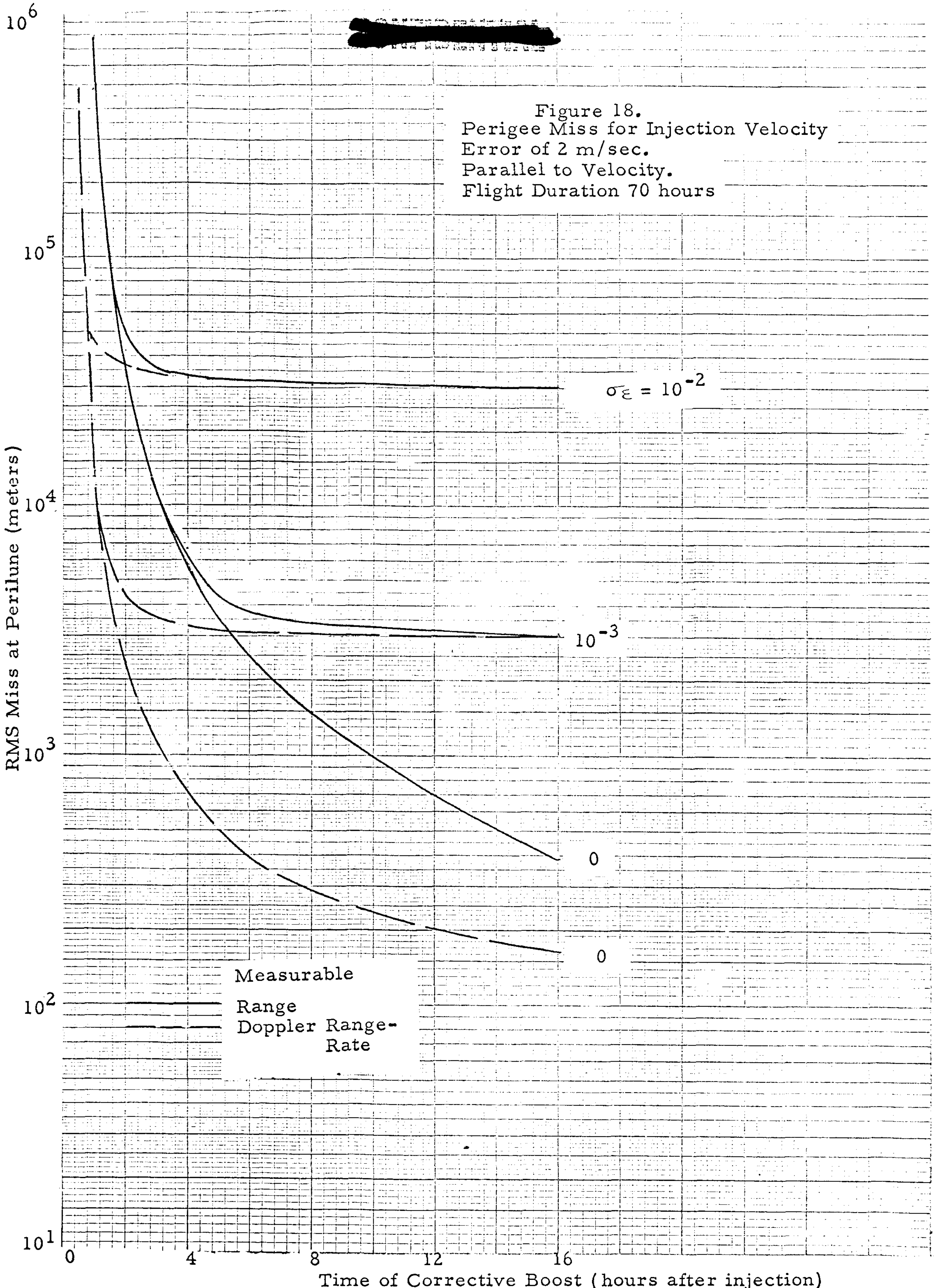

Figure 18.
Perigee Miss for Injection Velocity Error of 2 m/sec.
Parallel to Velocity.
Flight Duration 70 hours

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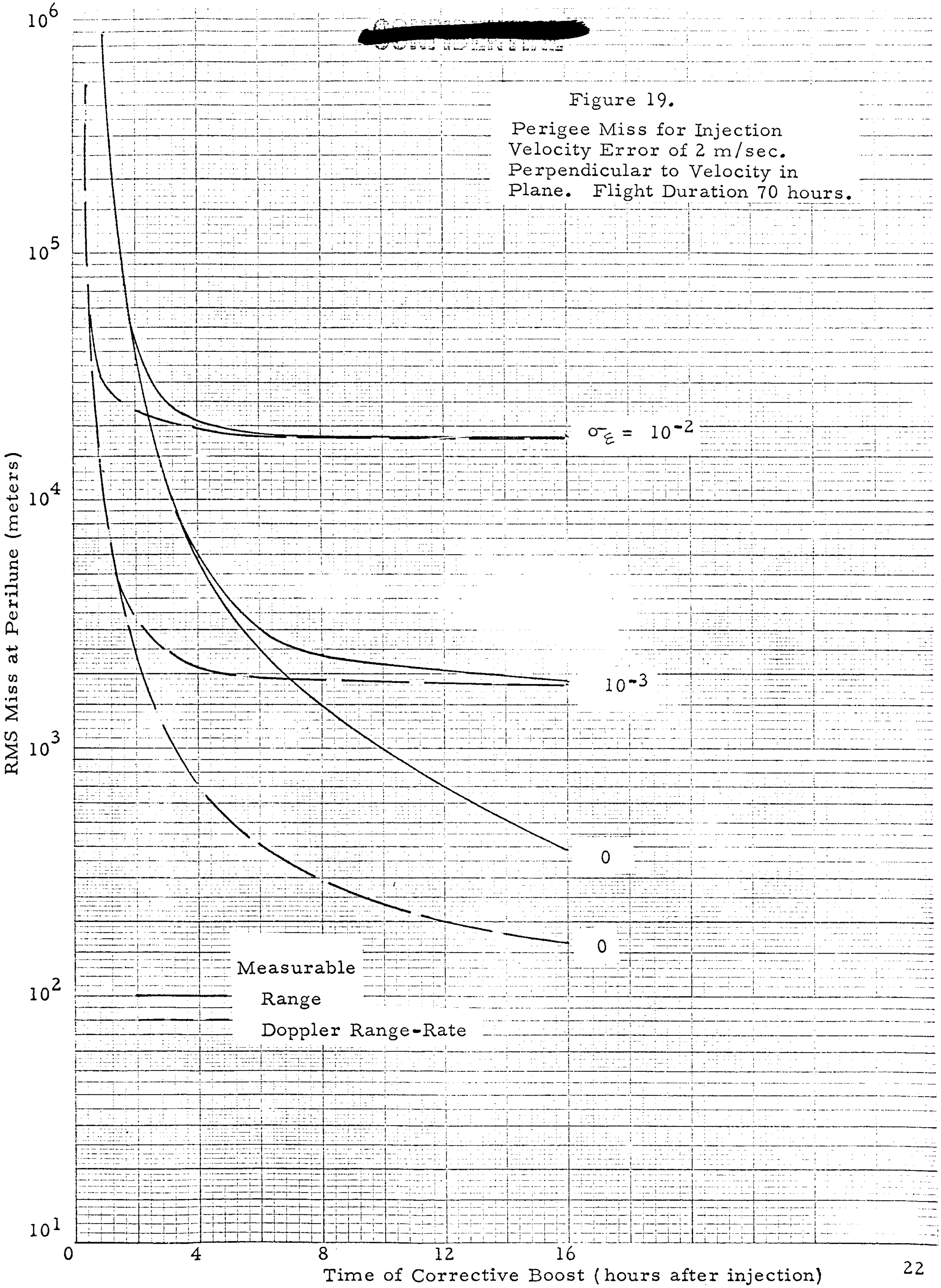

Figure 19.
Perigee Miss for Injection Velocity Error of 2 m/sec. Perpendicular to Velocity in Plane. Flight Duration 70 hours.

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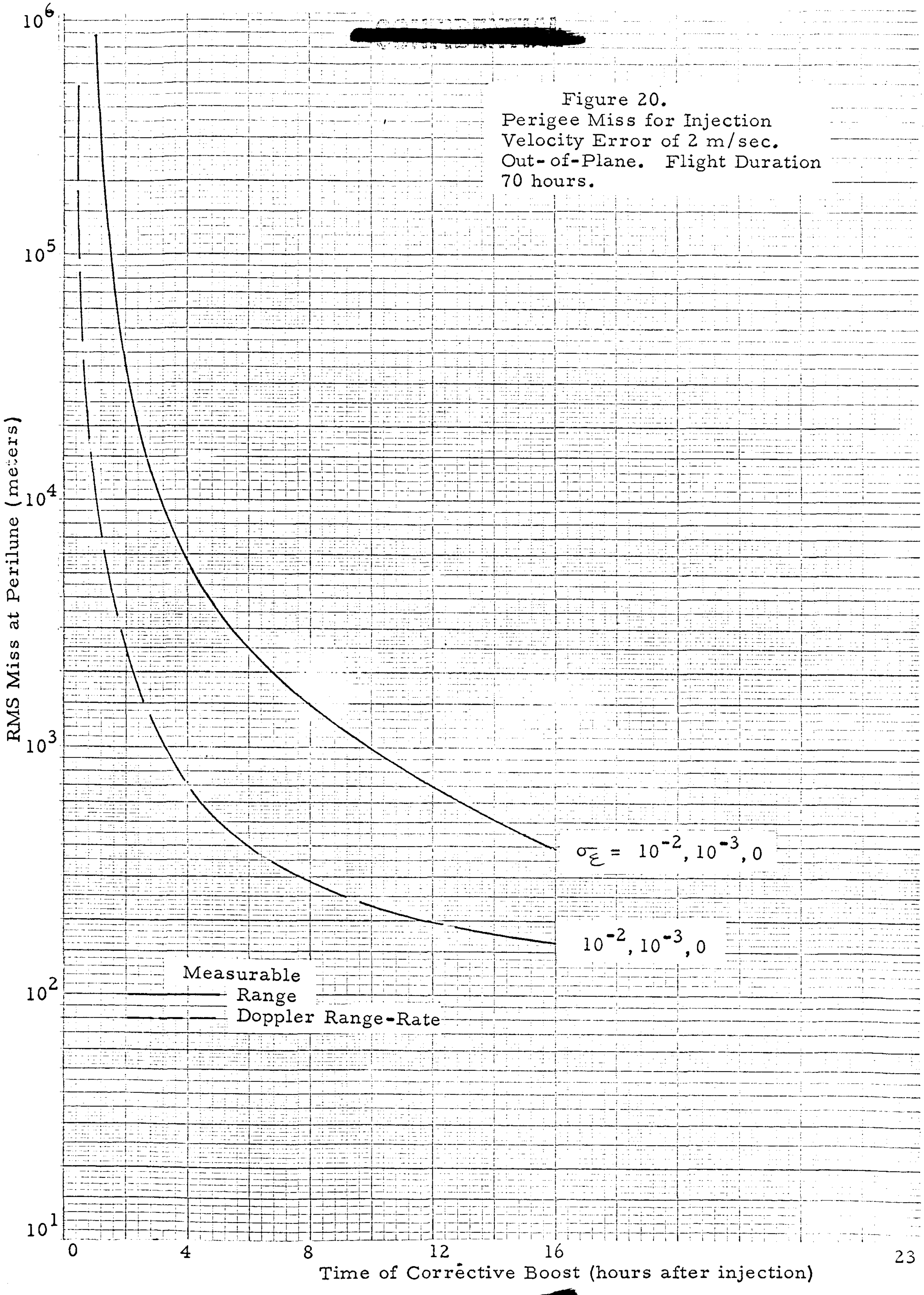

Figure 20.
Perigee Miss for Injection Velocity Error of 2 m/sec. Out-of-Plane. Flight Duration 70 hours.

[redacted]

[redacted]

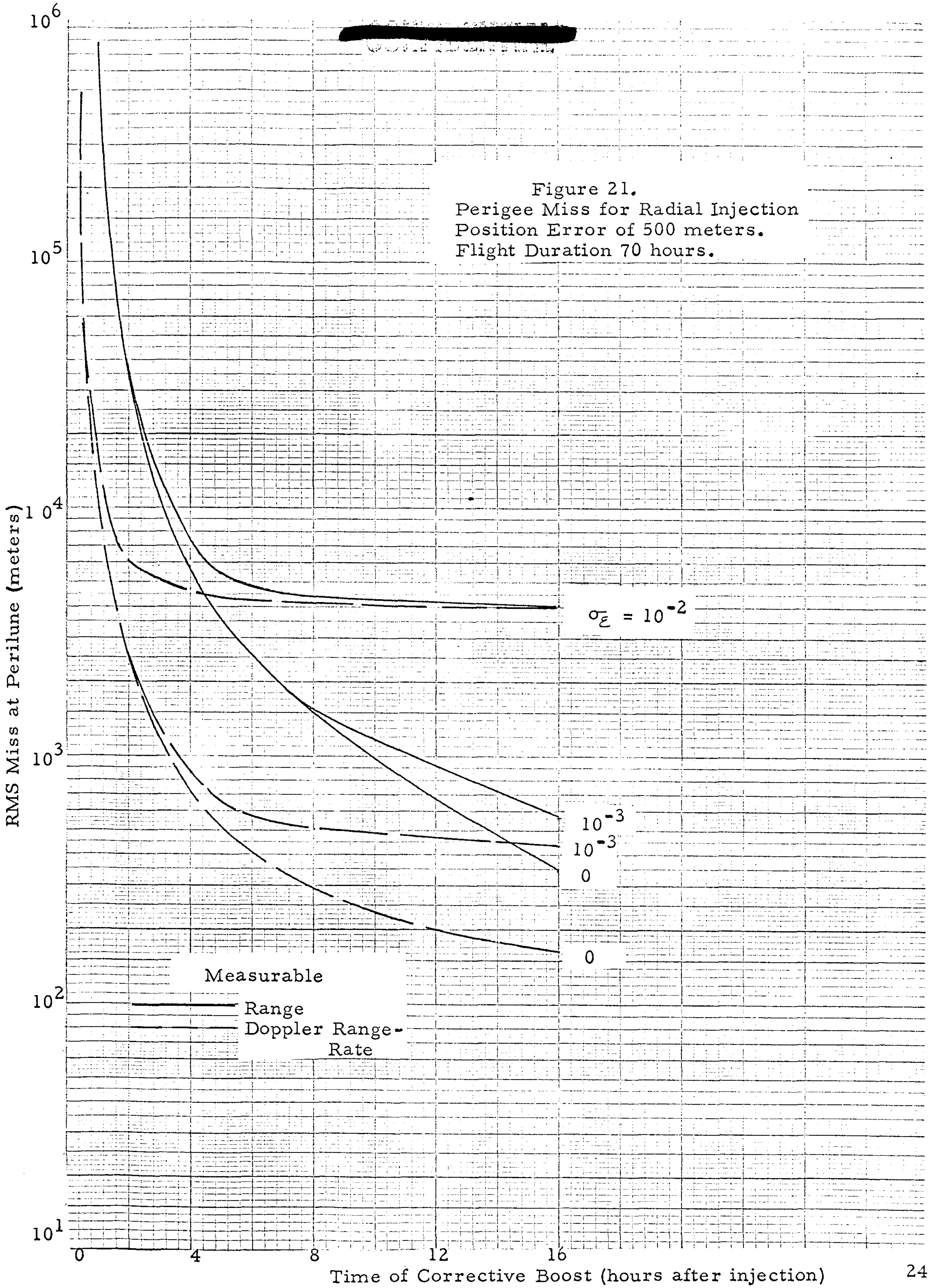

Figure 21.
Perigee Miss for Radial Injection Position Error of 500 meters. Flight Duration 70 hours.

[redacted]

[redacted]

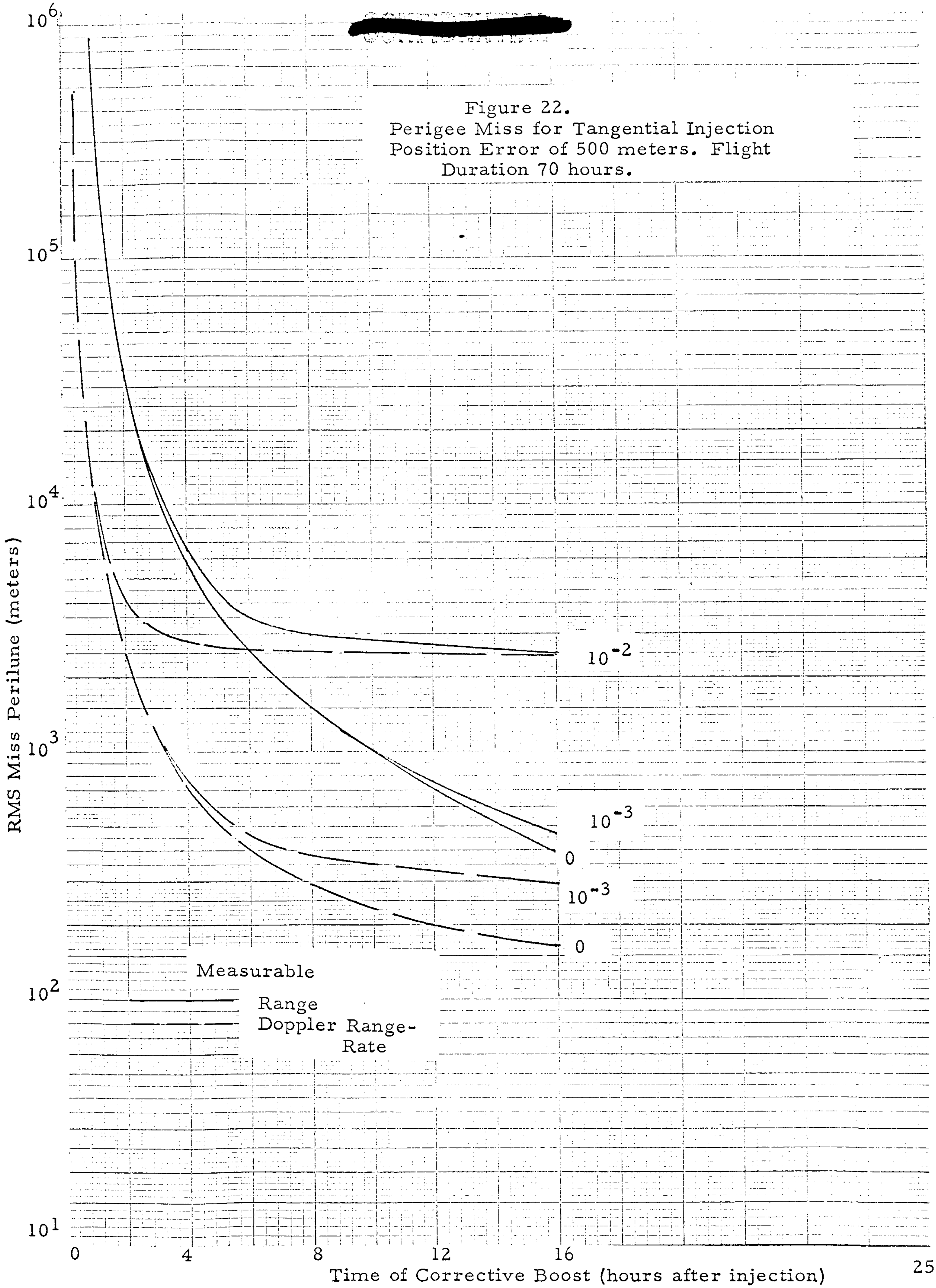

Figure 22.
Perigee Miss for Tangential Injection Position Error of 500 meters. Flight Duration 70 hours.

[redacted]

[redacted]

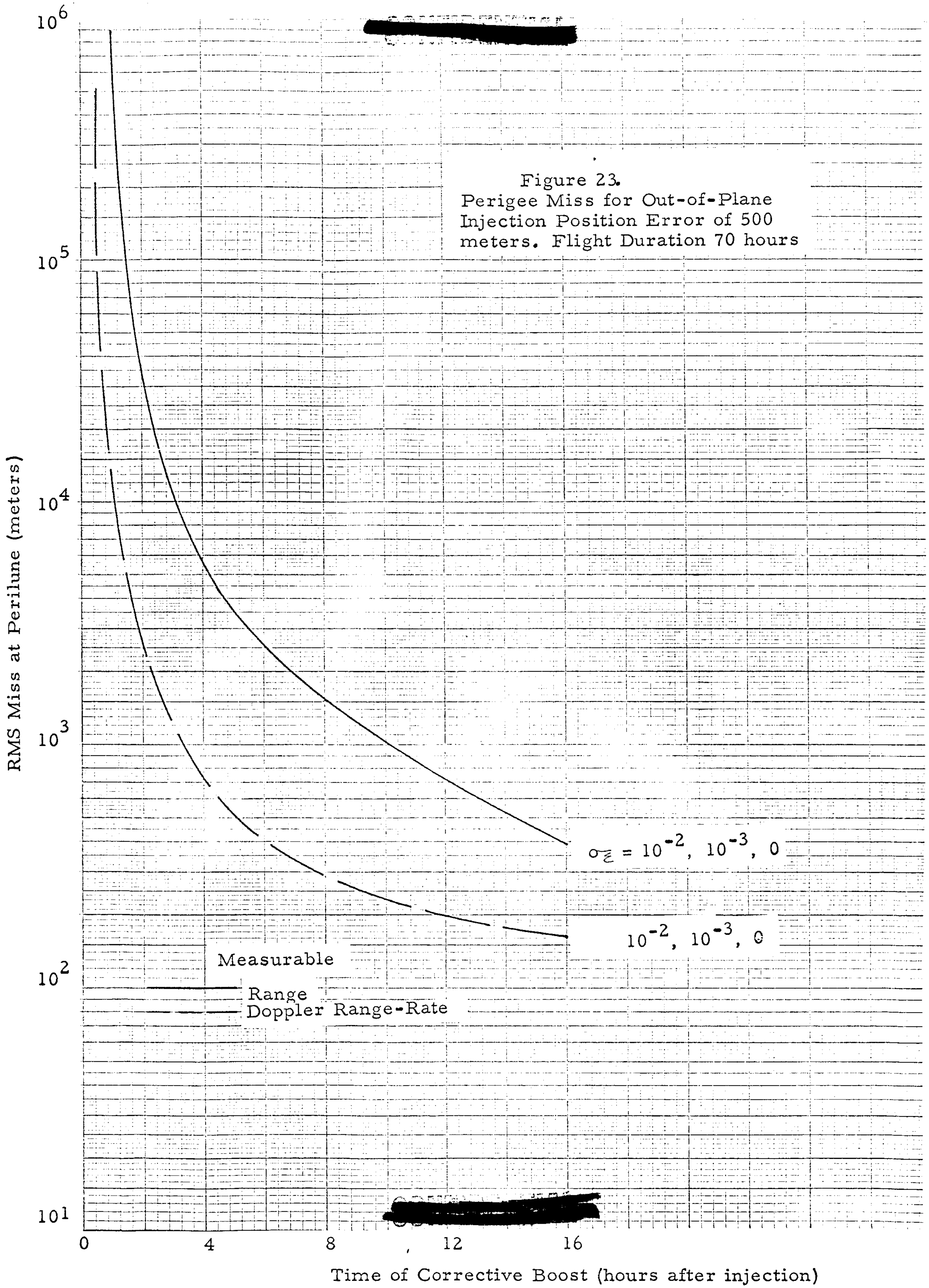

Figure 23. Perigee Miss for Out-of-Plane Injection Position Error of 500 meters. Flight Duration 70 hours

[redacted]

[redacted]

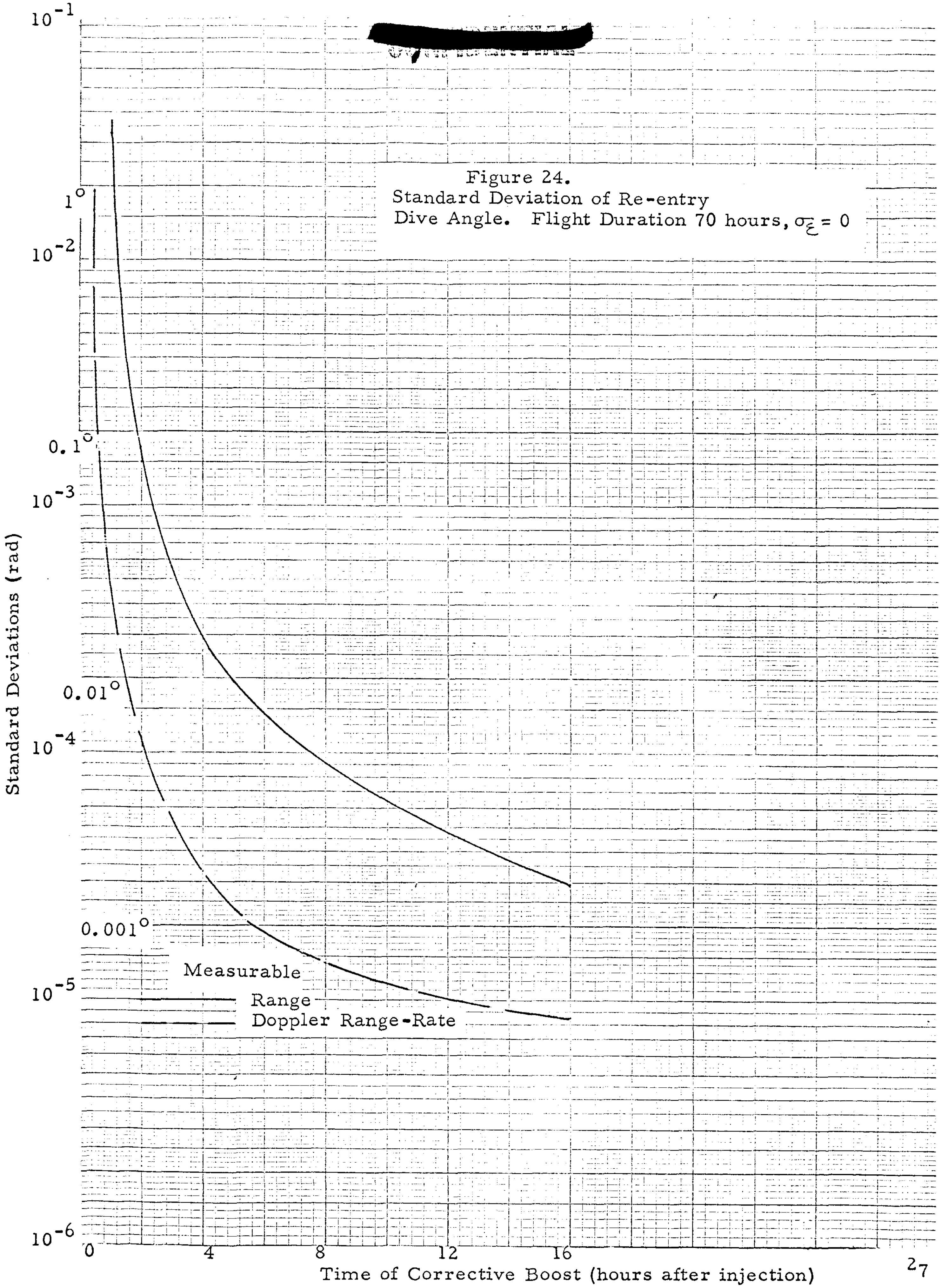

Figure 24.
Standard Deviation of Re-entry Dive Angle. Flight Duration 70 hours, $\sigma_{\xi} = 0$

[redacted]

Somewhat surprisingly, the perigee miss after the first corrective boost, with no execution error, is not a monotonic function of the duration of the Moon to Earth flight. For a corrective boost performed at any time, however, the variation in perigee miss as a function of flight duration is not more than a few thousand meters, and is not very important.

The expected value of error in re-entry angle varies by a negligible amount with flight duration. The same is true of the standard deviation of the re-entry angle for zero boost execution error. As in Apollo Note No. 270, the standard deviation of the re-entry angle far exceeds the expected value. The variation of the standard deviation of the re-entry angle with flight duration is small, and decreases with increasing flight duration.

The magnitude of the expected value of the first corrective boost is greatest for injection velocity errors along the velocity vector and for radial injection position errors. In both cases the required corrective boost has an initial rapid increase with the time between injection and the corrective boost, making it most economical to perform the first corrective boost early. Somewhat surprisingly, the expected magnitude of the corrective boost at a fixed time after injection increases slightly with increasing flight duration. This increase is not large.

Doppler range rate measurements from three MSFN stations with standard deviations of 0.1 cm/sec. for one minute observations provide considerable more accurate estimates of position and velocity than do range measurements once a minute from these same stations with standard deviations of 15 meters, and hence lead to smaller misses at perigee for zero boost execution error.

For finite boost execution errors (σ_{ε} equal to 10^{-3} or 10^{-2}), the higher accuracy of the Doppler system is useful only for the first hour or two after injection; after that the perigee miss depends on the boost execution error rather than MSFN accuracy. The main advantage of the higher Doppler accuracy is that it permits the first corrective boost to be performed at an earlier time, with a saving in boost cost.

[redacted]

The Bissett-Berman Corporation 2941 Nebraska Avenue, Santa Monica, California EXbrook 4-3270

APOLLO NOTE NO. C-4
(Task 3, Item III)

H. Engel
12 October 1964

OPTIMUM CORRECTIVE BOOST PROGRAM, V

Computations have been performed for single corrective boosts on translunar flights of varying duration, using range as the measurable. For flight times of 70 through 110 hours, the misses at perilune and the corrective boost magnitudes various times of application of the corrective boost are presented, for radial errors in injection position and for injection velocity errors along the injection velocity; these two kinds of injection errors have previously proved (Apollo Note No. 260) to require the greatest corrective boosts and to result in the greatest misses at perilune after the corrective boost.

Further, for completeness, the corrective boosts and misses at perilune are presented for times of translunar flight of 70 hours for injection position errors in the tangential and out-of-plane directions, and for injection velocity errors perpendicular to the injection velocity in the plane of flight and orthogonal to the plane of flight.

Still further, the magnitudes of the corrective boosts and the resultant misses at perilune are shown for a translunar flight of 70 hours, for injection position errors in each of three orthogonal directions and for injection velocity errors in each of three orthogonal directions using Doppler range rate as the measurable.

The results are presented and conclusions drawn.

Cases

In order to obtain a family of varying time translunar flights, we have made some simplifying assumptions that make the computations easier without changing the conclusions that can be drawn from consideration of this family of flights.

The two principal simplifying assumptions are that the Moon rotates about the Earth in a circular orbit in the plane of the Earth's equator, and that the flights are from perigee to perilune. The two assumptions make conic patching simpler, and do not greatly change the ability of MSFN stations to estimate space vehicle positions and velocities.

Figure 1 shows the RMS miss at perilune after the first corrective boost for an injection error of 10m/sec. along the velocity vector, an execution error σ_{ε} of 10^{-3}, and various times of flight, all for range measurements of 15m each minute from each of three MSFN stations.

Figure 2 presents similar results for a radial injection position error of 5000m.

Figures 3 and 4 show the corresponding expected values of corrective boost.

It will be noted that in Figures 1 through 4, the results for flight durations of 90 and 110 hours are partially or totally missing. We have encountered some difficulties in performing the necessary calculations for these cases. Rather than withholding this note until the source of these difficulties has been tracked down, we are presenting the remaining results and are relying upon the regular behavior of miss and required boost as functions of flight duration exhibited in Apollo Note No. 272.

It will be observed that, as in Apollo Note No. 260, the best time to perform the corrective boost is as early as possible, (one hour after injection), although this does not result in the smallest miss at perilune, the cost of a second boost to eliminate this remaining miss being very small. It should also be observed that if the corrective boost is performed at the recommended time, the expected value of the corrective boost does not depend on the time of flight for flight times less than 100 hours.

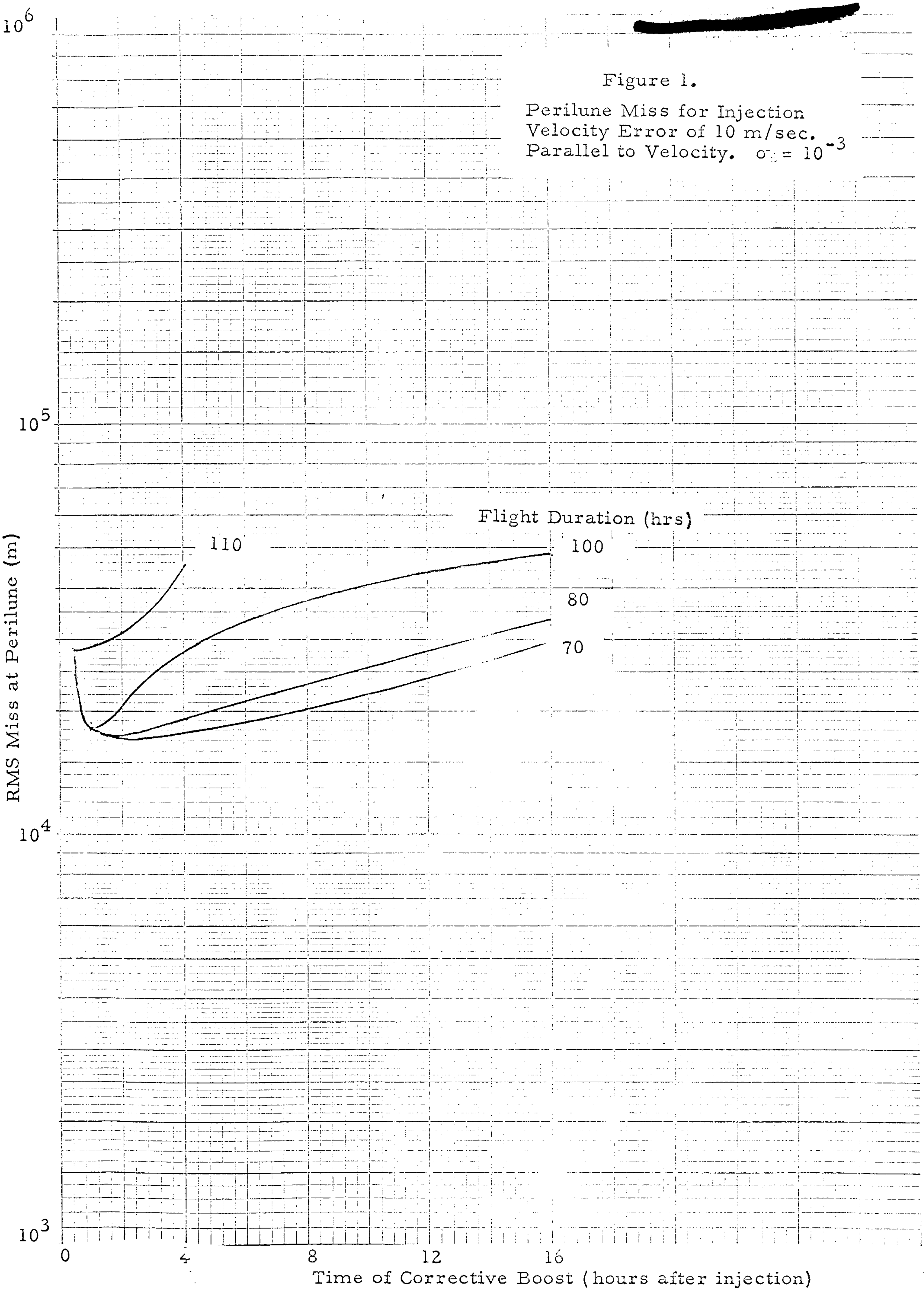

Figure 1.

Perilune Miss for Injection Velocity Error of 10 m/sec. Parallel to Velocity. $\sigma = 10^{-3}$

[redacted]

[redacted]

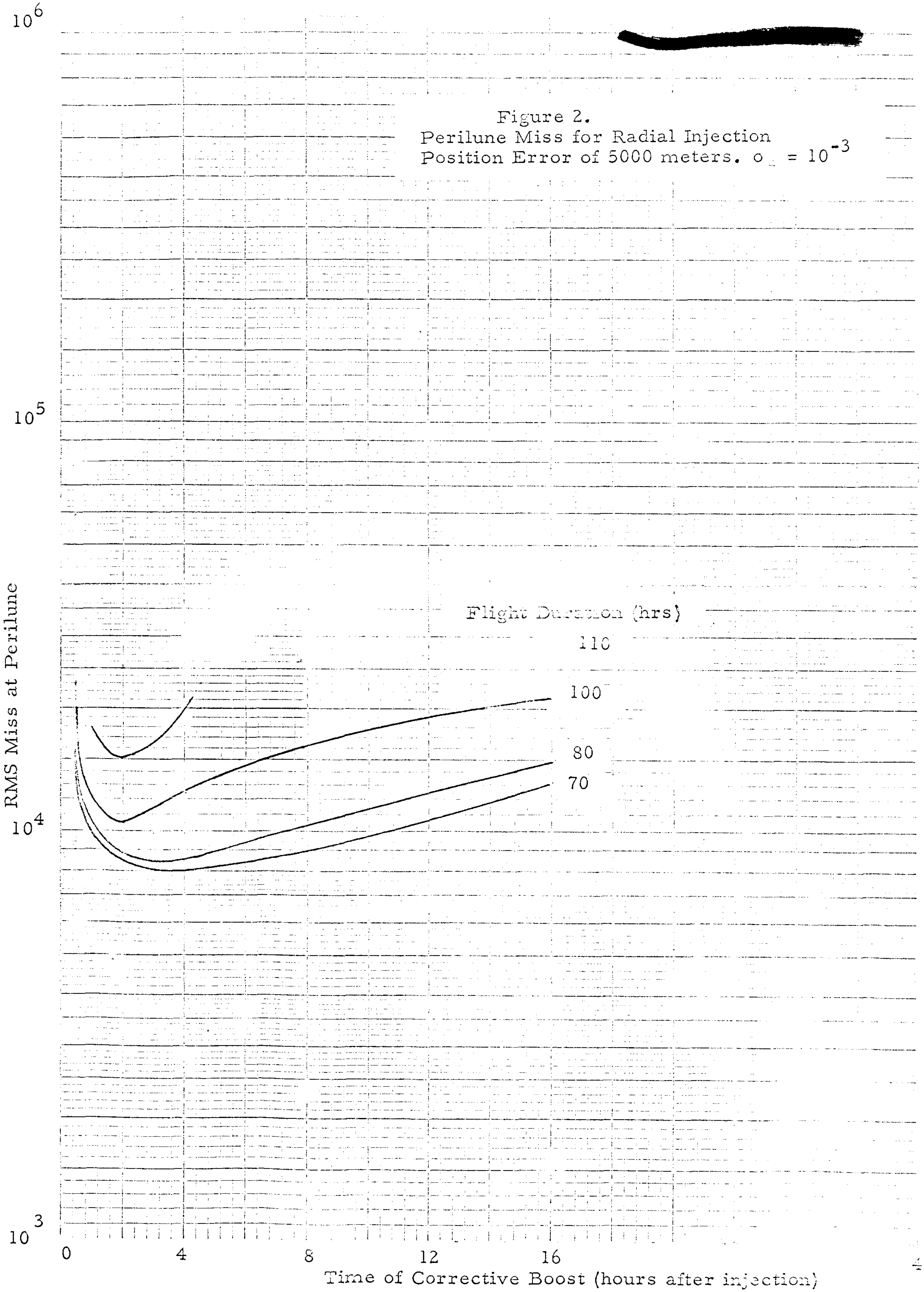

Figure 2.
Perilune Miss for Radial Injection
Position Error of 5000 meters. $\sigma = 10^{-3}$

[redacted]

[redacted]

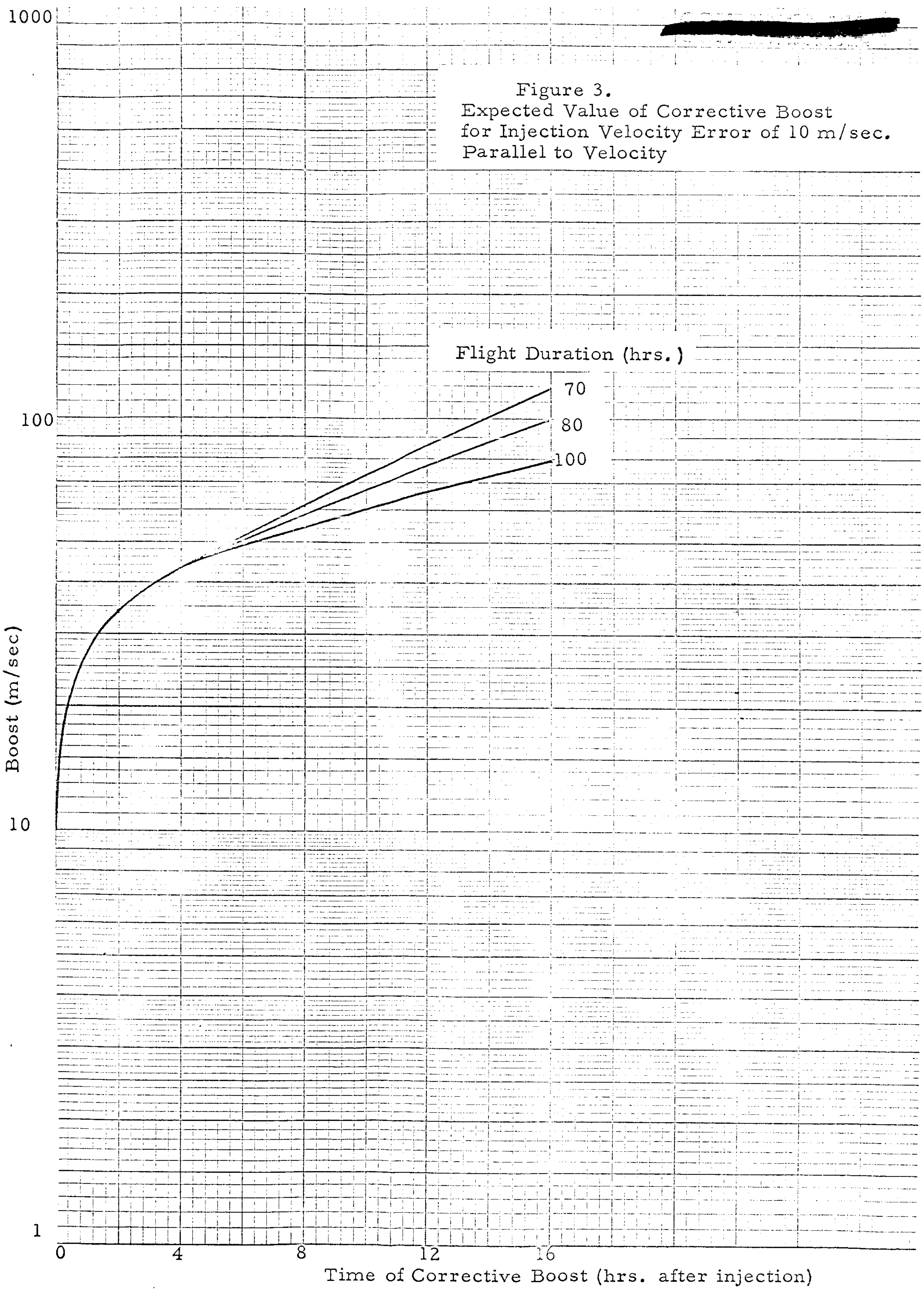

Figure 3.
Expected Value of Corrective Boost for Injection Velocity Error of 10 m/sec. Parallel to Velocity

[redacted]

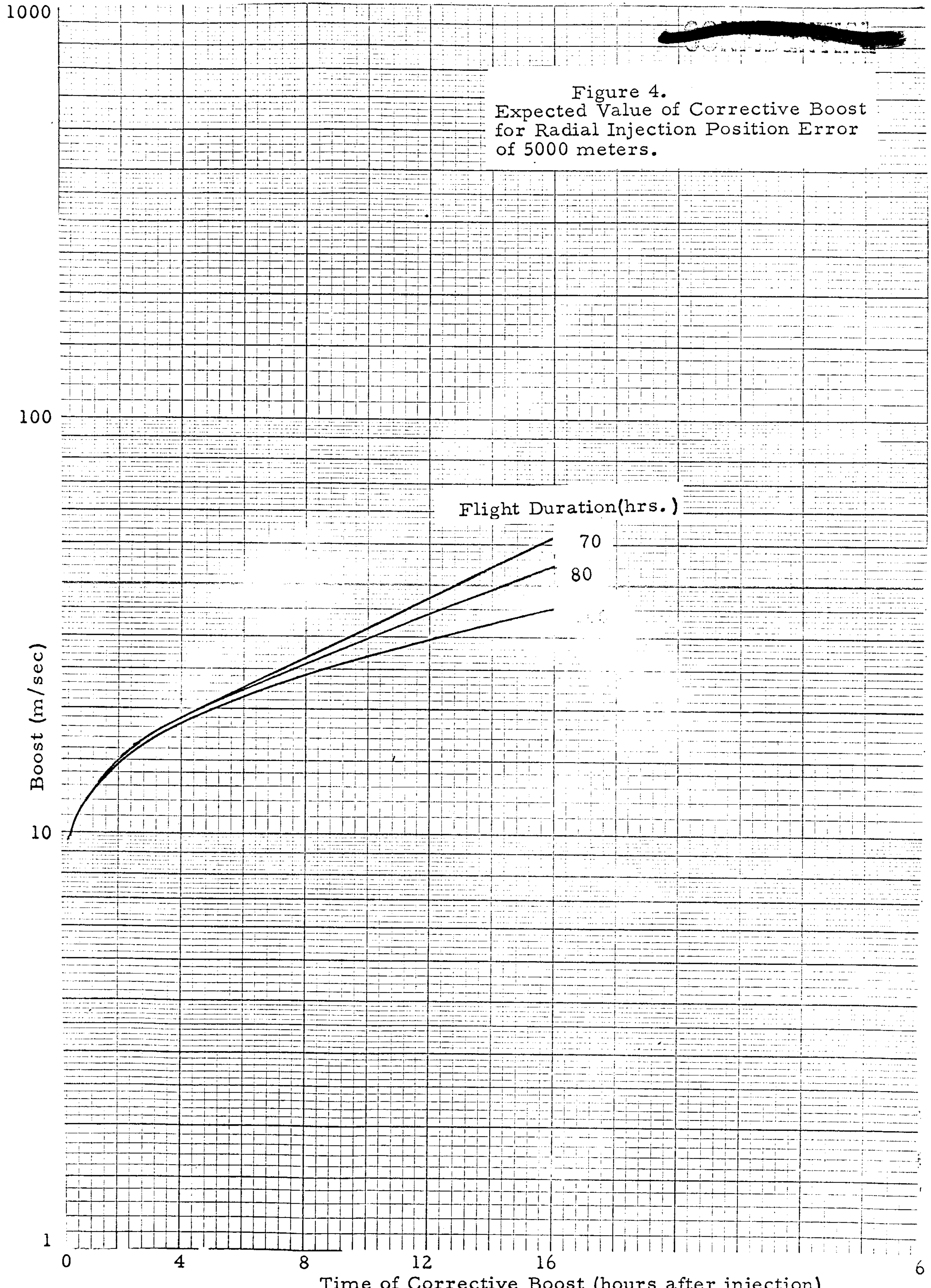

Figure 4.
Expected Value of Corrective Boost for Radial Injection Position Error of 5000 meters.

If the corrective boost is performed at a later time, then the magnitude of the expected value of the corrective boost appears to decrease if the time of flight is increased. If the corrective boost is performed at a later time, the RMS miss at perilune after the corrective boost increases with the time of flight, but this is not an important consideration since in any case this remaining miss can be eliminated by means of a very small second corrective boost.

In Figures 5 through 10 the dashed lines show the RMS miss at perilune for a time of flight of 70.18 hours for injection velocity errors of 10 meters/sec. along the velocity vector, perpendicular to the velocity vector in the plane of flight, and out-of-the-plane of flight, and for injection position errors of 5000m radially, tangentially in the plane of flight and out-of-the-plane of flight, all for range observations with a standard deviation of 15 meters obtained from each of three MSFN station each minute. In these same figures the dashed lines represent the RMS miss at perilune if Doppler measurements from the three MSFN stations are used instead, with one master station and two slave stations, Doppler velocity being measured with a standard deviation of 0.1 cm/sec. each minute. These results are shown for execution errors σ_{ϵ} of 0, 10^{-3} and 10^{-2}.

As in Apollo Note No. 260, injection velocity errors along the velocity vector and radial injection position errors result in the greatest RMS miss at perilune after the corrective boost.

Although the Doppler measurements provide more accurate estimates of position and velocity than do the range measurements, as indicated by the curves for σ_{ϵ} equal to 0 in Figures 5 through 10, it turns out that for execution errors σ_{ϵ} of the order of 10^{-3} or 10^{-2} the RMS miss at perilune is not appreciably reduced through use of these better estimates. The reason for this is that the major portion of the miss at perilune is due to errors in the performance of the commanded corrective boost rather than inaccuracy of knowledge of what this boost should be.

[redacted]

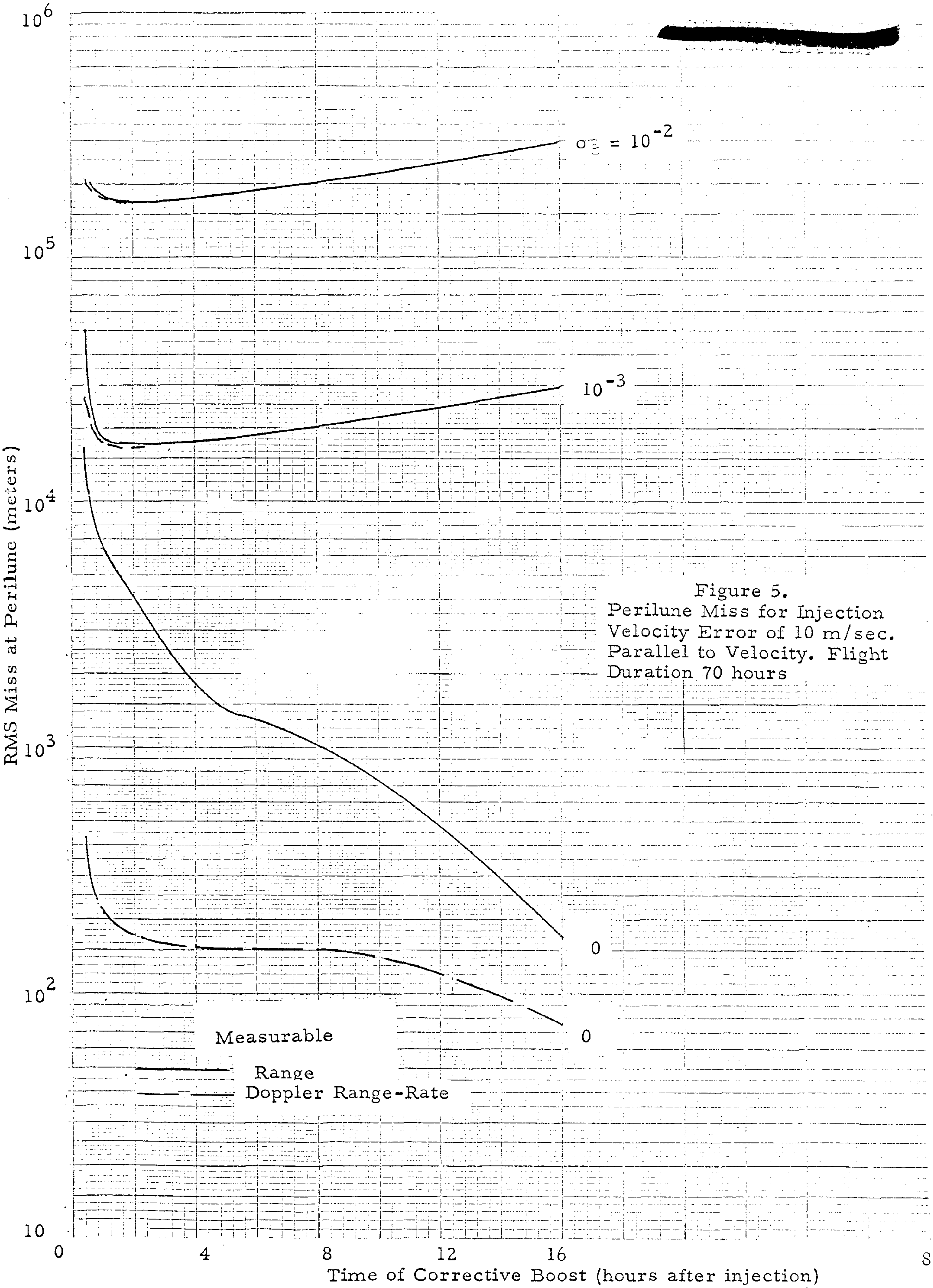

Figure 5.
Perilune Miss for Injection Velocity Error of 10 m/sec. Parallel to Velocity. Flight Duration 70 hours

[redacted]

[redacted]

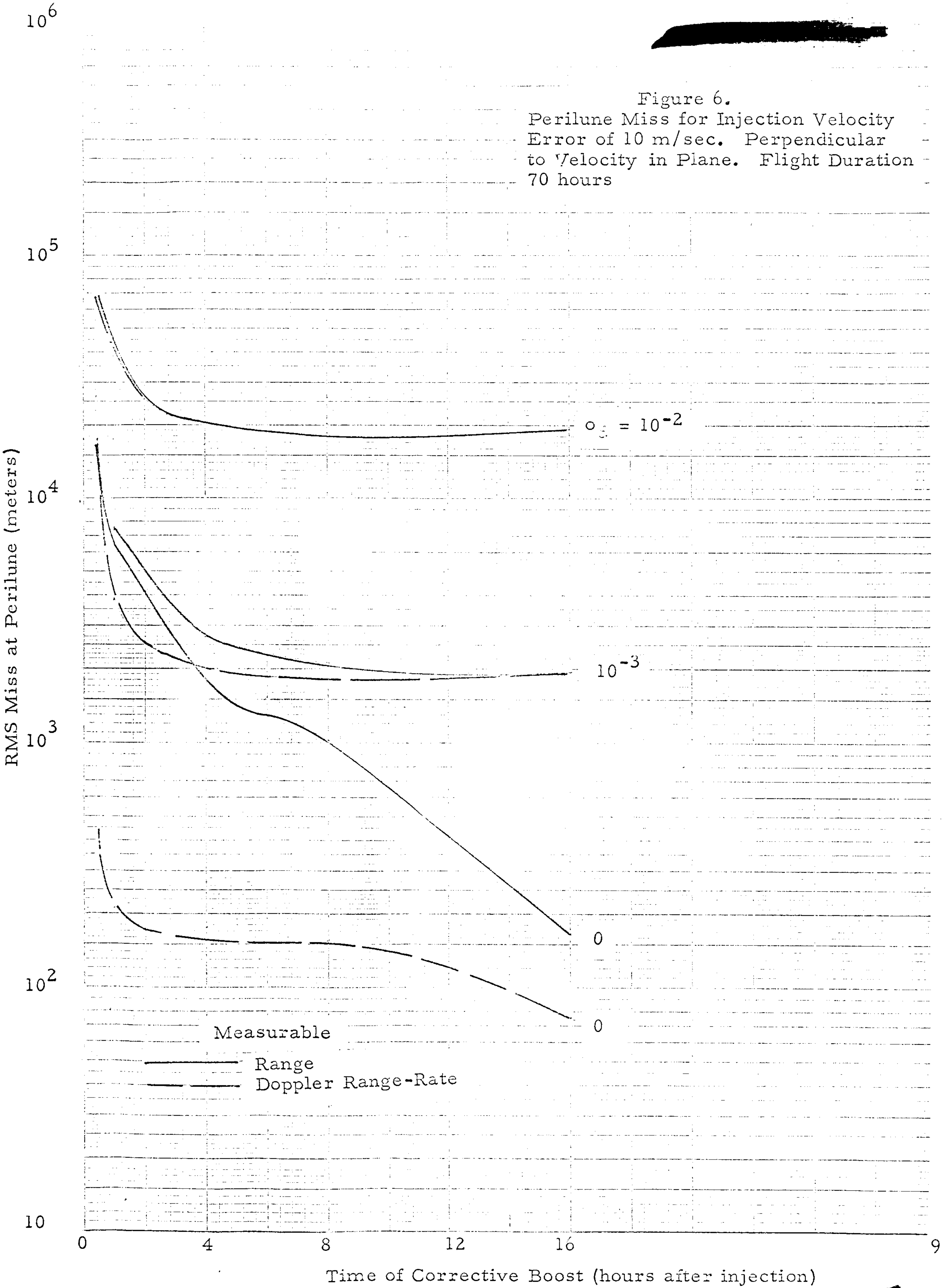

Figure 6.
Perilune Miss for Injection Velocity Error of 10 m/sec. Perpendicular to Velocity in Plane. Flight Duration 70 hours

[redacted]

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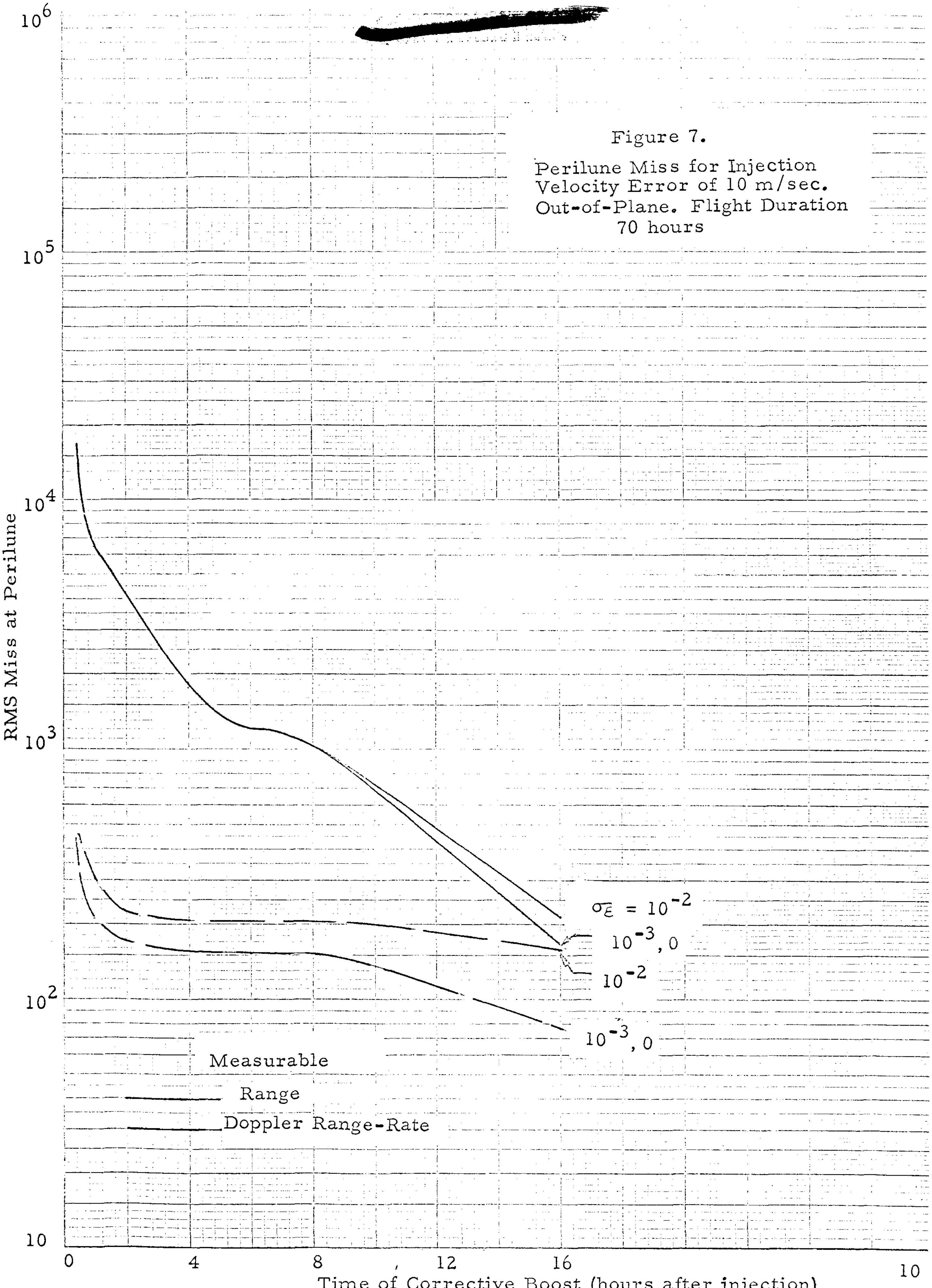

Figure 7.

Perilune Miss for Injection Velocity Error of 10 m/sec. Out-of-Plane. Flight Duration 70 hours

[redacted]

[redacted]

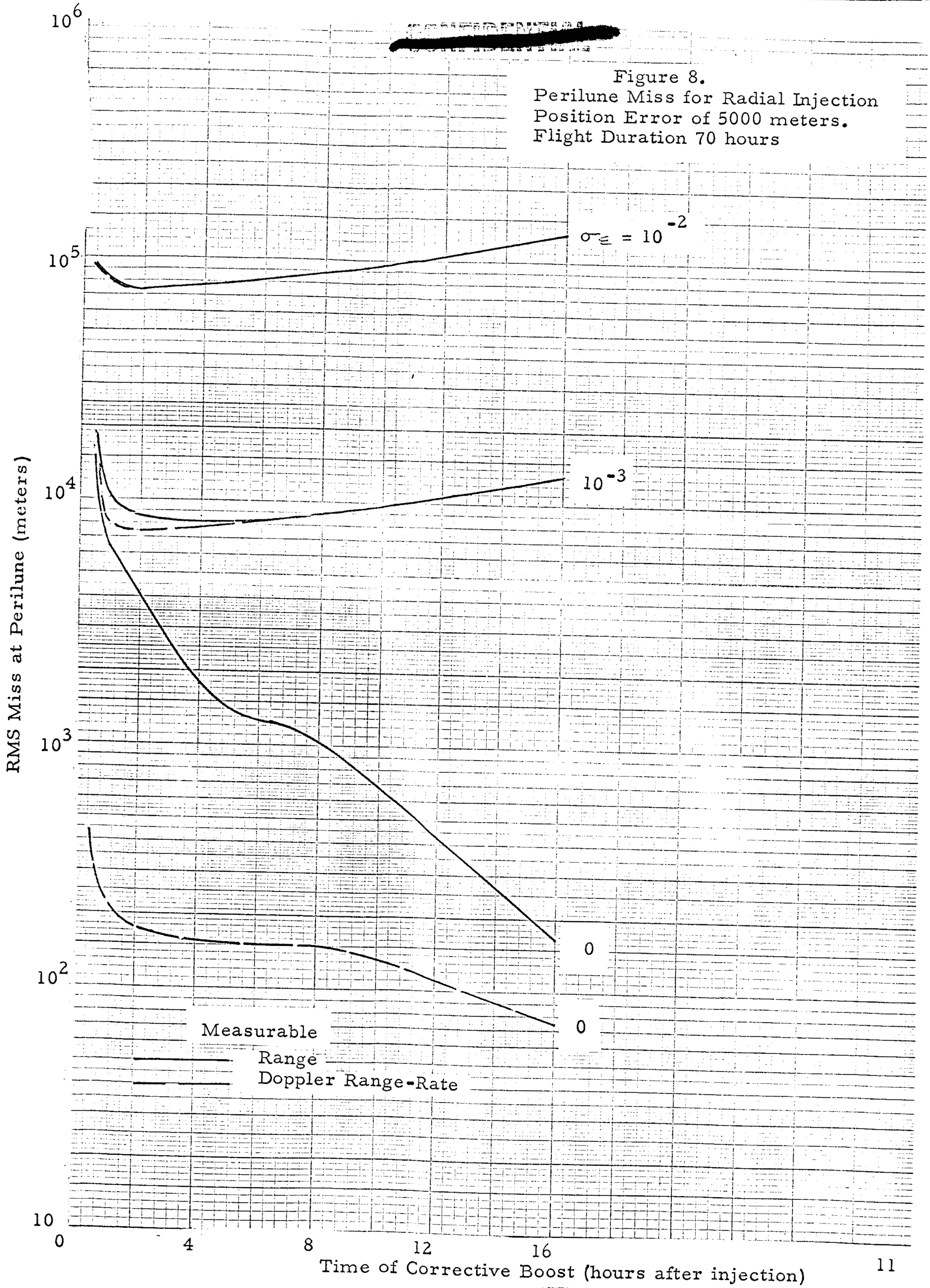

Figure 8.
Perilune Miss for Radial Injection Position Error of 5000 meters. Flight Duration 70 hours

[redacted]

[redacted]

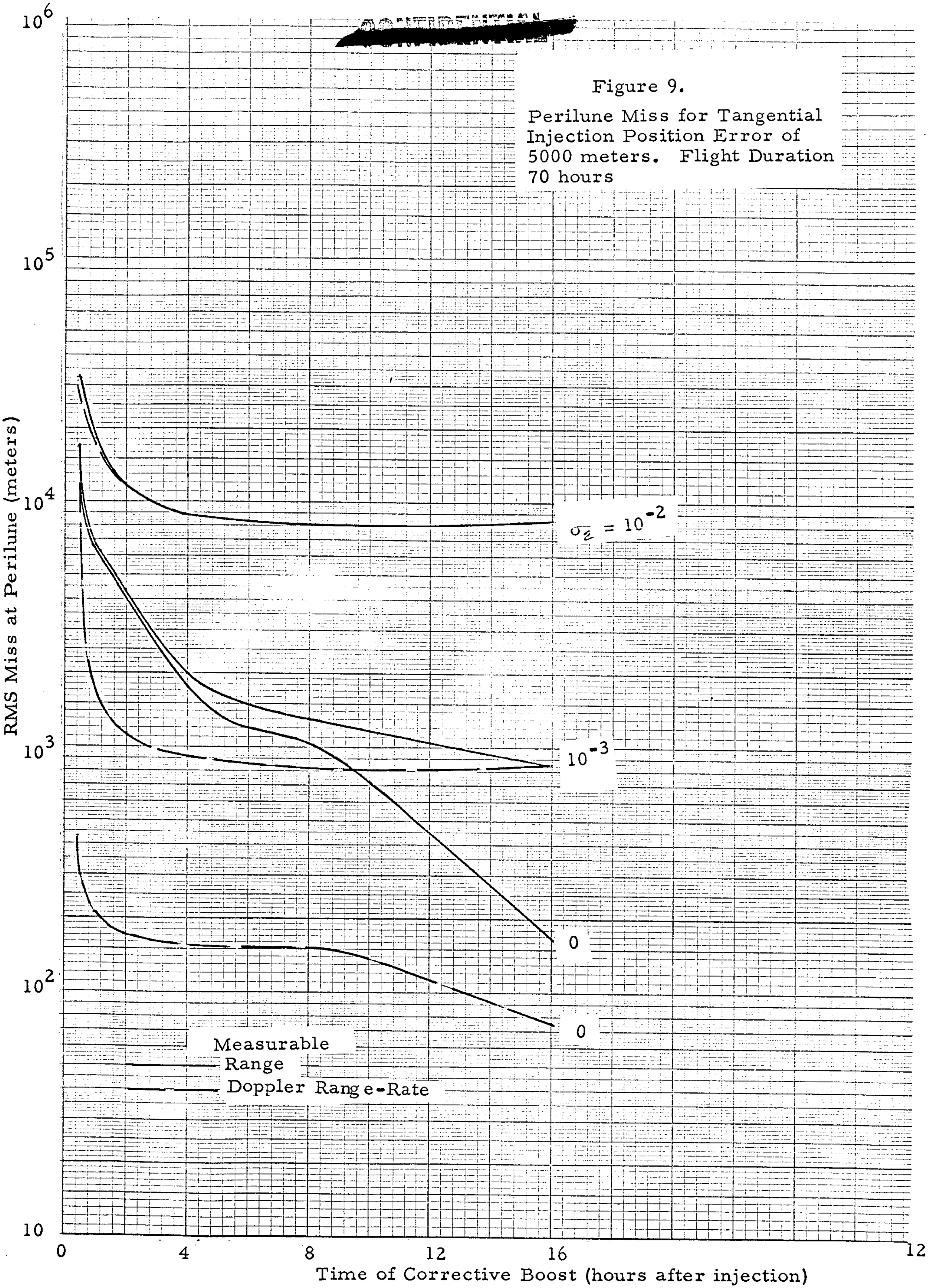

Figure 9.

Perilune Miss for Tangential Injection Position Error of 5000 meters. Flight Duration 70 hours

[redacted]

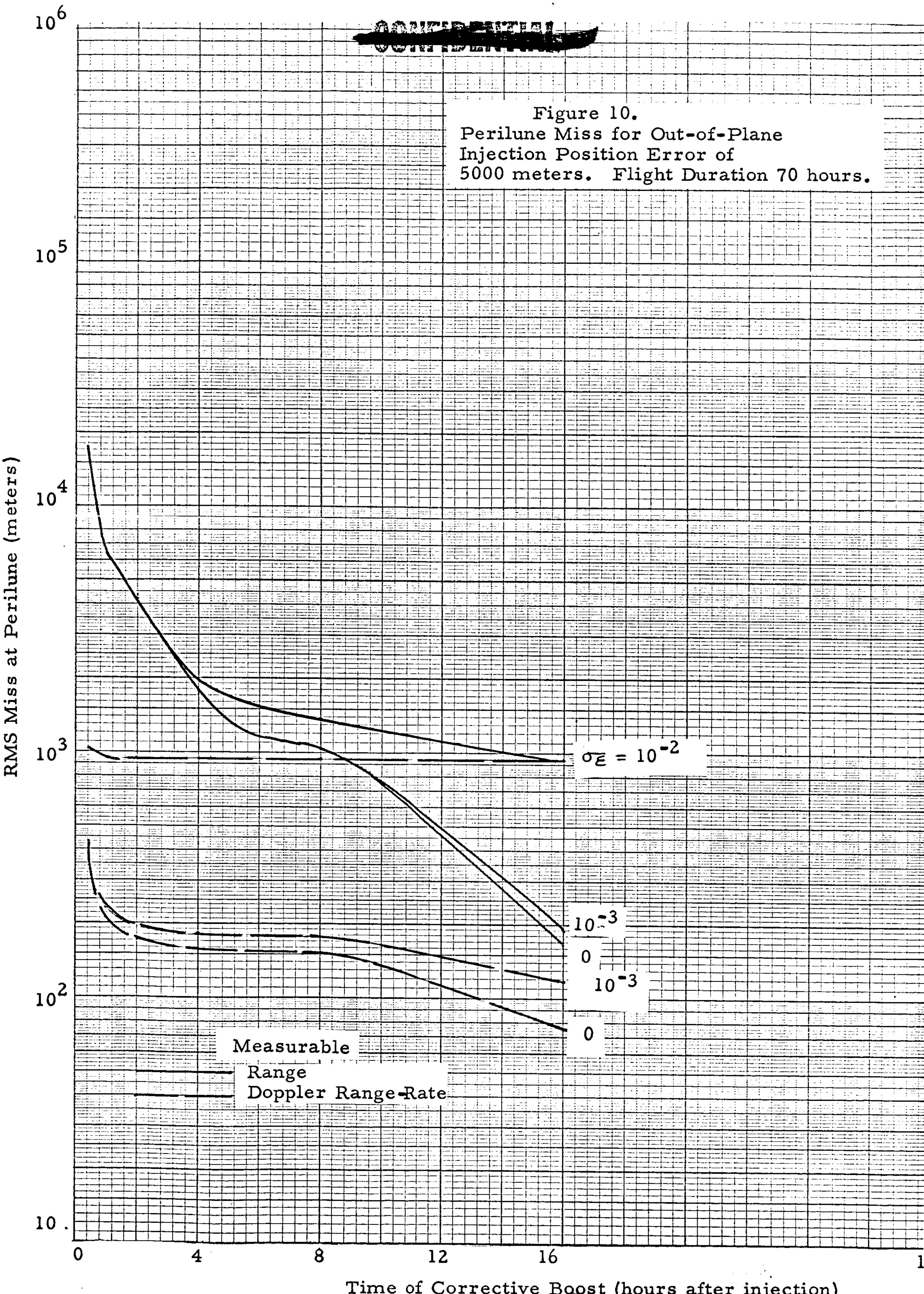

Figure 10.
Perilune Miss for Out-of-Plane Injection Position Error of 5000 meters. Flight Duration 70 hours.

The reader may observe that in some of the Figures 5 through 10 there is apparently a substantial difference in the RMS miss at perilune depending on whether range or Doppler information is used, but the reader should remember that the largest misses after the corrective boost result from injection velocity errors along the velocity vector and from radial injection position errors, and note that for these injection errors the statement made above holds.

The expected value of the corrective boost is independent of the execution error σ_{ε} and is greatest for injection velocity errors parallel to injection velocity and for radial injection position errors. These largest boosts are shown in Figures 3 and 4 for all times of flight considered.

The standard deviation in the corrective boost and the error in velocity at perilune are both small for corrective boosts performed early, as indicated in Apollo Note No. 260, and so are not shown in this note.

It should be noted that it is possible to scale the results presented in this note and in Apollo Note No. 260 to account for various magnitudes of injection errors. The required corrective boost varies directly as the magnitude of the injection error. The RMS miss at perilune increases proportionally if the injection error and the execution error are increased proportionally together.

Conclusions

For the varying time translunar flights all the conclusions of Apollo Note No. 260 still hold.

Further, as the time of flight increases the perilune miss remaining after the corrective boost increases and the required corrective boost decreases.

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APOLLO NOTE NO. C-5
(Task 3, Item III)

H. Engel
19 October 1964

OPTIMUM CORRECTIVE BOOST PROGRAM, VI

This note presents the results of computations for the AMPTF translunar reference mission with translunar injection performed on the first, second or third Earth orbit, and with either the MIT or MSFC guidance and navigation systems employed from lift-off through injection. In all cases ground radars are used to determine position and velocity prior to injection.

The position and velocity errors at the end of injection burn are given in Table 1. They are based on 3 σ-values of errors in the on-board navigation system and on 1 σ errors for the ground radars.

Whether injection occurs on the first, second or third orbit the same translunar trajectory has been employed for these calculations. This should not greatly affect the results, and greatly reduces the amount of calculation necessary.

If the orbit parameters determined by one ground radar are used as a priori values for the orbit parameters calculated by the next, then the estimates of position and velocity prior to injection improve steadily. In this case, using either the MIT or MSFC navigation system, as shown in Figures 1 and 2, it is better to wait for two or three orbits before injecting since the smaller errors before injection result in smaller injection errors.

This effect is more noticeable with the MIT navigation system, since the astronauts are able to realign the IMU prior to injection.

[redacted]

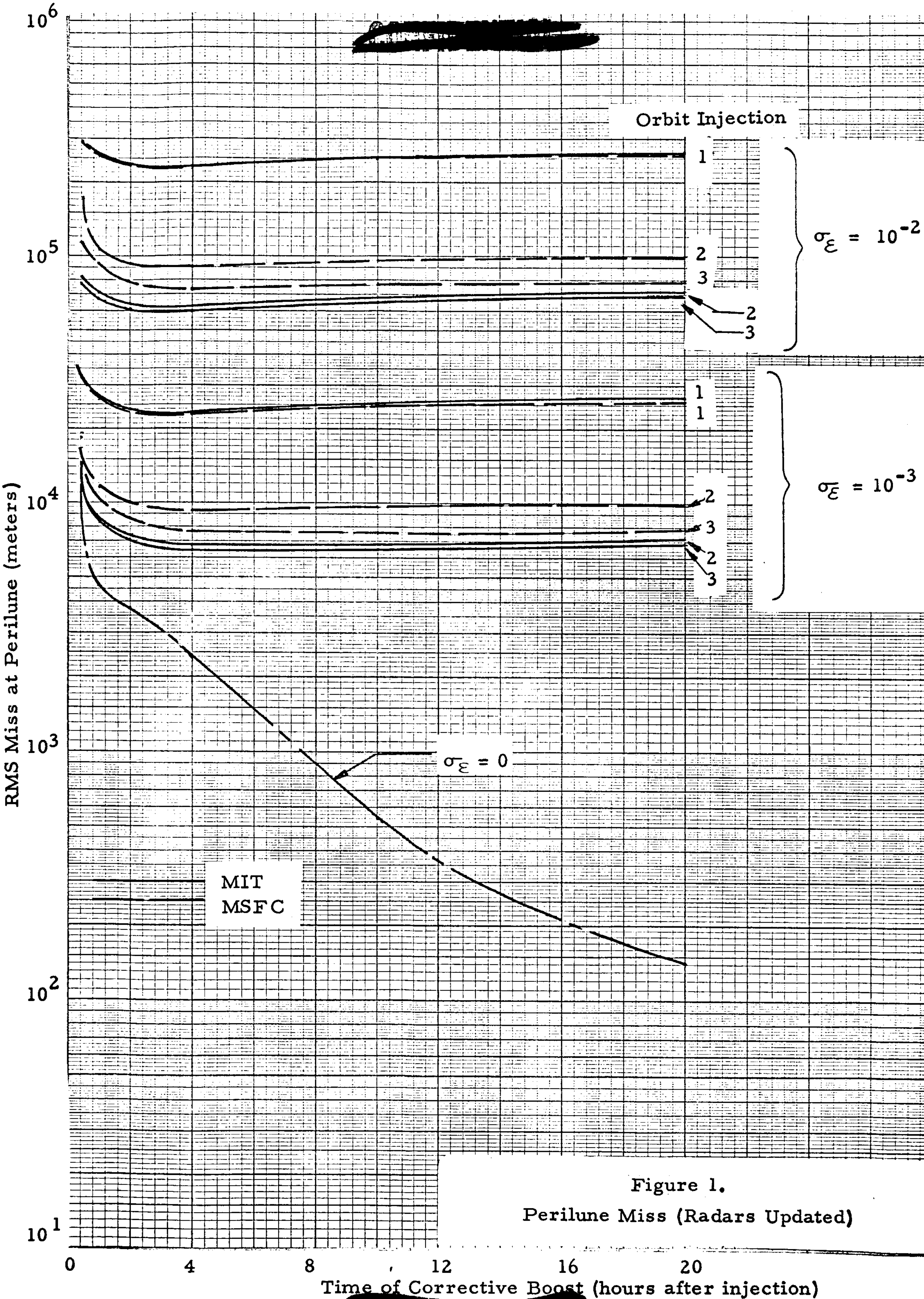

Figure 1.
Perilune Miss (Radars Updated)

[redacted]

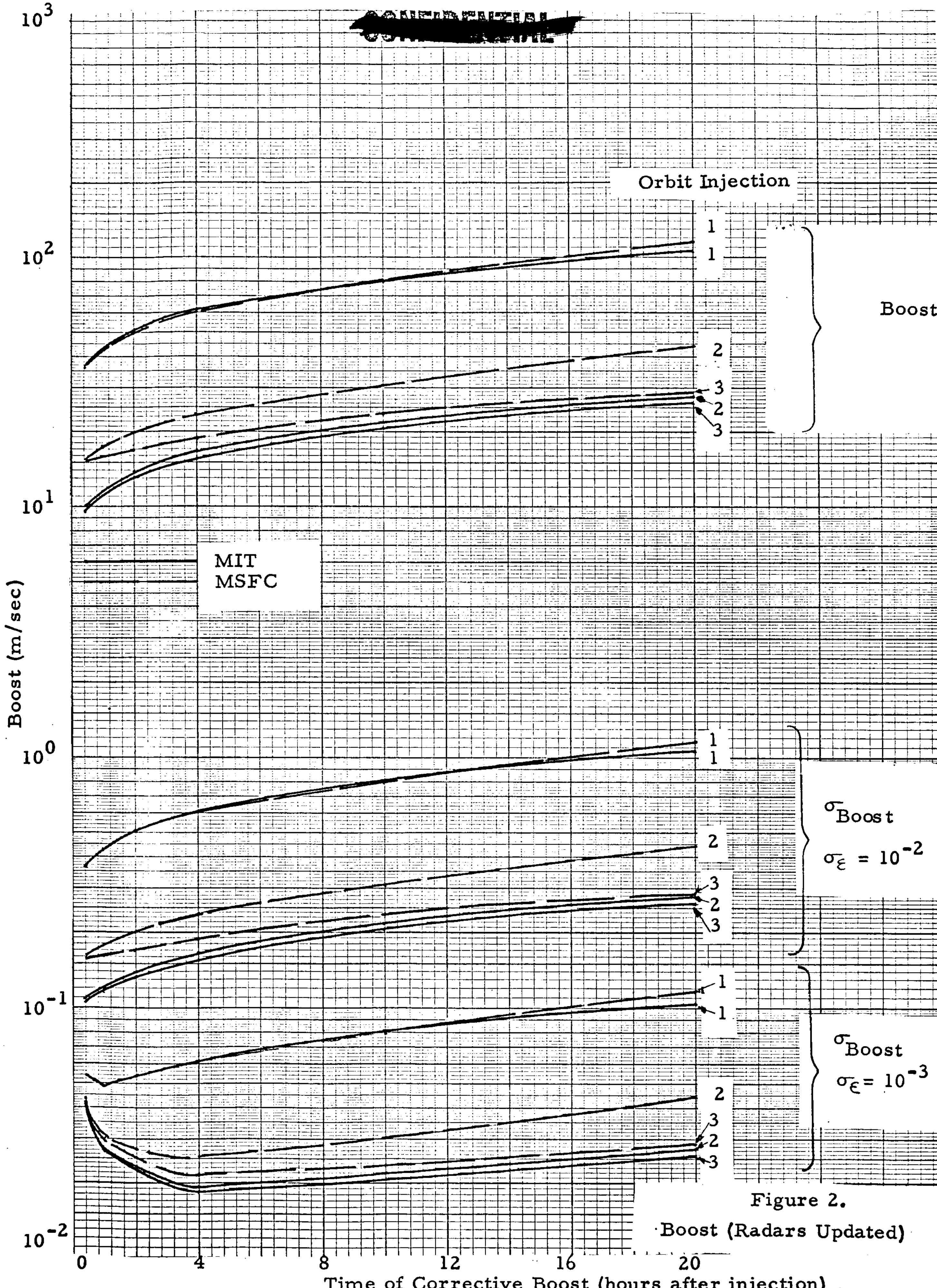

Figure 2.
Boost (Radars Updated)

Table 1.

RMS Values of Components of Position and Velocity Errors Prior to Injection Boost

Orbit Number	Position (meters)	Velocity (m/sec)
1	10^3	10
2	10^2	1
3	10^2	0.2

For the MIT system, the injection errors on the first orbit are due primarily to errors in position and velocity prior to injection; in the second and third, or subsequent orbits, the injection errors are due primarily to the on-board navigation system and do not vary with the orbit in which injection occurs.

The reduction of injection errors with increasing time before injection is not as great with the MSFC navigation system because the gyros cannot be realigned after lift-off. In fact, if we examined injection errors after a larger number of orbits the MSFC injection errors would eventually increase.

The situation is different if the orbit parameters determined by one ground radar are not used as a priori values by the next radar. In this case, illustrated in Figures 3 and 4, the injection errors are about the same on any orbit when using the MIT system. In this case, if the MSFC system is used the injection errors are smallest on the first orbit, and grow on subsequent orbits because of drift of the gyros. Note that in Figures 3 and 4 the pre-injection position and velocity errors have been specified arbitrarily, and not determined by the error analysis program.

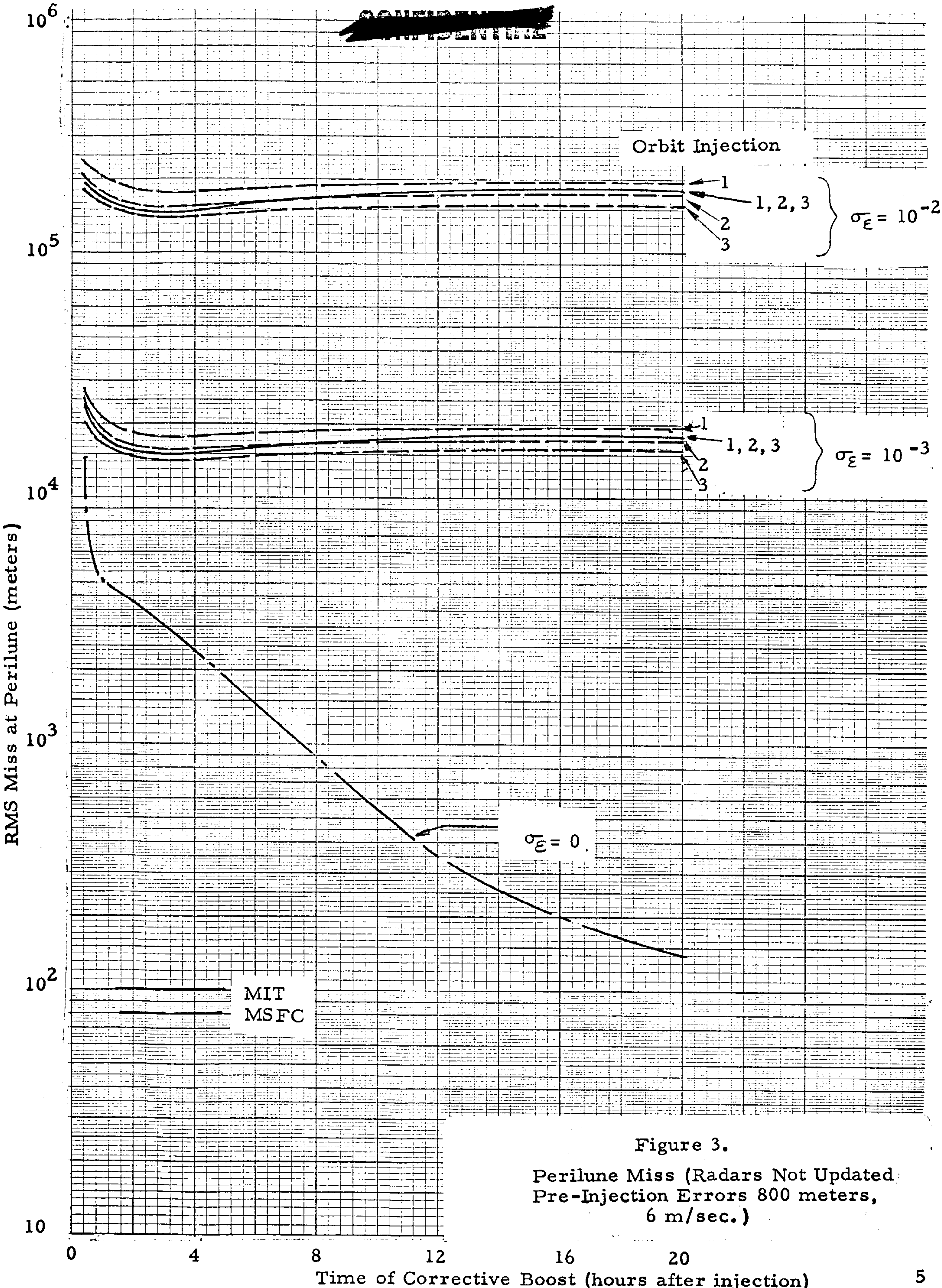

Figure 3.

Perilune Miss (Radars Not Updated Pre-Injection Errors 800 meters, 6 m/sec.)

[redacted]

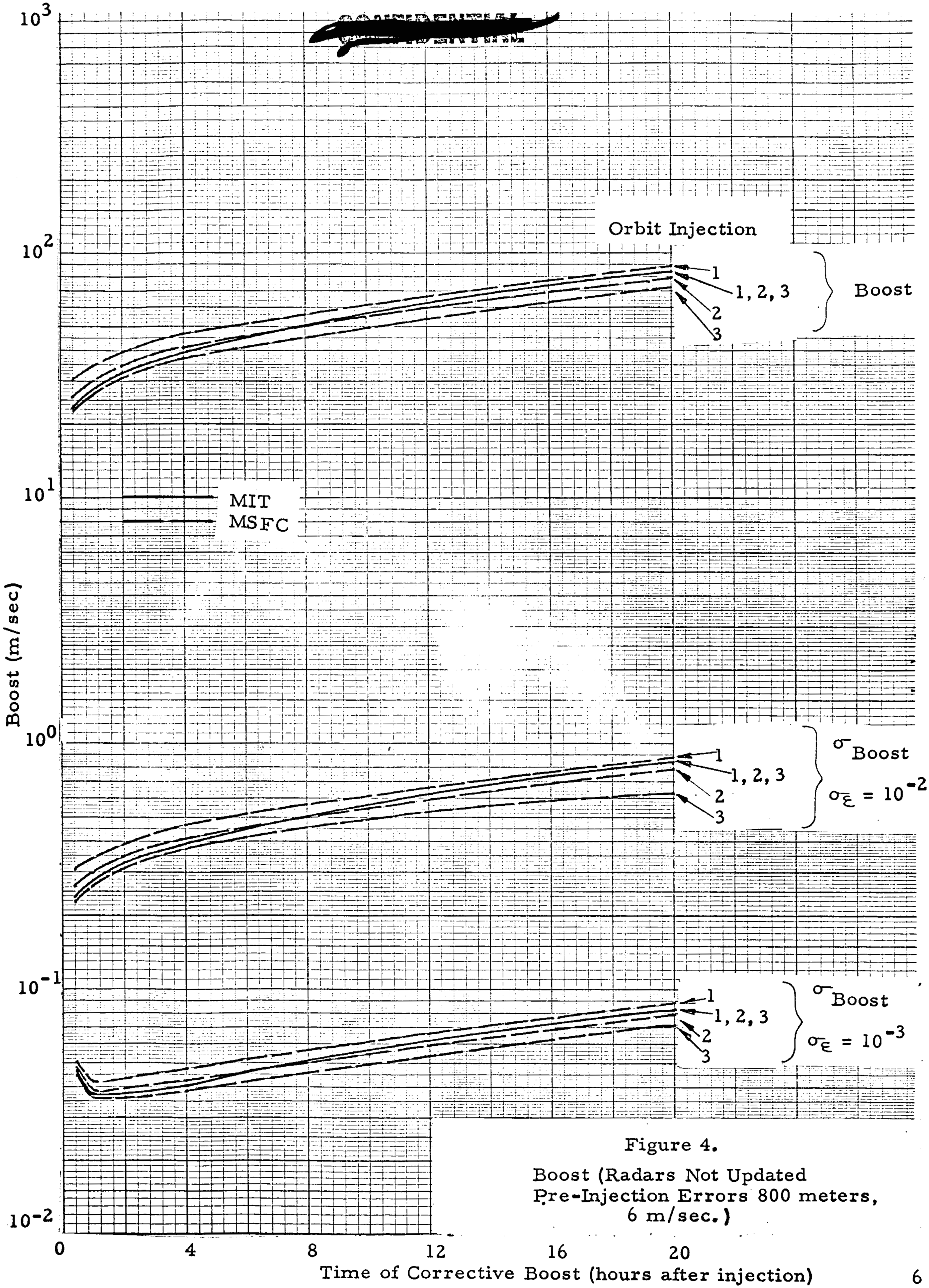

Figure 4.

Boost (Radars Not Updated Pre-Injection Errors 800 meters, 6 m/sec.)

[redacted]

Examination of the results shows that there is a very substantial reduction in the amount of corrective boost required if the orbit parameters determined by one station are used as a priori data by the next. Thus, it is worth extra effort to perform these computations. The extra effort primarily involves estimating the venting thrust; the air drag effects are small.

Let us consider the air drag first. The weight in parking orbit is about 280,000 pounds. Assuming a 20 foot diameter for the SIV-B and assuming the vehicle is aligned with the velocity vector, the ballistic coefficient is

$$\frac{W}{C_D S} \doteq \frac{280,000}{2 \times \frac{\pi}{4} \times 20^2} = 445.$$

At 100 miles altitude, the air density is about 10^{-12} slugs/ft^3, so the acceleration due to drag in a circular orbit is:

$$\frac{g \frac{\rho}{2} V^2}{\left(\frac{W}{C_D S}\right)} = \frac{32 \frac{10^{-12}}{2} (25,900)^2}{445} = 2.4 \times 10^{-5} \text{ ft/sec}^2$$

In half an orbit this would result in a change in velocity of the order of 0.06 ft/sec. and a change in position of the order of 72 feet, ignoring the central force field. Even if the air drag correction were totally ignored, these errors would be negligible in using the orbit determined by one radar as a priori for the next.

The venting acceleration in Earth orbit has previously been estimated at 3 ft/sec. per orbit, leading to a velocity change of the order of 1.5 ft/sec. and a position change of 1800 feet in half an orbit (0.5 m/sec. and 600 meters). From this it is apparent that if continuous venting along the longitudinal axis of the vehicle is used, this venting must be estimated as an orbit parameter if the orbit determined by one radar is to be used as a priori data by the next. Alternatively a non-propulsive venting system such as that suggested by Douglas Aircraft Company and pictured on page 55 of the October 5, 1964 issue of Aviation Week must be employed.

Another point of interest is a comparison between the results given in these Apollo Notes on the Optimum Corrective Boost Program, in which the guidance rule is to reduce the calculated perilune miss to zero with each corrective boost, and the results that might be expected if the MIT guidance rule were employed instead. In the MIT guidance rule the first boost is used to reduce the calculated miss to zero at approximately the lunar sphere of influence. The advantages of the MIT guidance rule are that it reduces the velocity error at perilune (which has already been shown to be very small) to a smaller value, and that the amount of computation required may be less. On the other hand, the MIT guidance rule results in a small increase in the cost of the corrective boosts. The reason for this is that using the MIT guidance rule the initial injection error must be wiped out in the time it takes to get to the LSOI, whereas in the guidance rule used in these notes we have until perilune to wipe out the injection error. In both cases, it will be best to perform the first corrective boost as soon as possible after injection. We do not feel that the differences in perilune miss or in boost costs are sufficient basis for making a choice between the two guidance rules.

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APOLLO NOTE NO. C-6
(Task 3, Item III)

H. Engel
23 October 1964

OPTIMUM CORRECTIVE BOOST PROGRAM, VII

This note presents the results of computations for the AMPTF translunar reference mission with translunar injection performed on the first, second or third Earth orbit. The errors in position and velocity at injection have been computed for a number of circumstances:

1. MSFC navigation system used alone.
2. MIT navigation system used alone.
3. MSFC navigation system used for injection boost, but pre-injection position and velocity determined by MSFN C-band radars.
4. MIT navigation system used for injection boost, but pre-injection position and velocity determined by MSFN C-band radars.

Still further, various accuracies have been employed for the MSFN determination of position and velocity prior to injection. These are:

1. 400 meters RMS error in position and 2 m/sec. RMS error in velocity, corresponding to present MSC 1 σ estimates immediately after a pass over a single MSFN station.
2. 800 meters RMS error in position and 6 m/sec. RMS error in velocity, corresponding to a pessimistic MSFN estimate of pre-injection position and velocity.
3. 1000 meters RMS error in position and 10 m/sec RMS error in velocity, corresponding to Bissett-Berman estimates for the first orbit without an insertion tracking ship.

4. 100 meters RMS error in position and 1 m/sec. RMS error in velocity, corresponding to Bissett-Berman estimates for the second orbit, using up-dating from station-to-station.

5. 100 meters RMS error in position and 0.2 m/sec. RMS error in velocity, corresponding to Bissett-Berman estimates for the third orbit, using up-dating from station-to-station.

The conditions prior to injection based on use of the MSFC navigation system only or the MIT navigation system only have been computed neglecting the effect of venting, which provides an additional acceleration of about 0.02 cm/sec^2. On the first, second and third orbits the resultant errors in the injection conditions through neglect of venting would be very roughly

Orbit	Velocity	Position
1	0.5 m/sec.	600 meters
2	1.5	5400
3	2.5	15000

The MSFC accelerometer 3 σ zero bias is 0.01 cm/sec^2, so that even if the accelerometers were employed during Earth parking orbit to measure the venting acceleration there would be a substantial increase in the injection errors on the second and third orbits. The MIT inertial navigation system performance when used for insertion and injection without updating from other data is so poor that the additional error due to venting is negligible. (It should be borne in mind that the MIT system was not designed or intended for this task).

It seems likely that the venting acceleration can be estimated or determined by flow measurements substantially more accurately than by the on-board inertial navigation system, so we have not included the effects of venting on the pre-injection position and velocity errors when the on-board systems are used to determine pre-injection position and velocity.

When the MSFN is used to determine pre-injection position and velocity without updating the errors due to totally neglecting venting will be of the order of 600 meters and 0.5 m/sec. or less because only a small portion of an orbit ensues between the last MSFN observation and injection. Even rough estimates of the venting acceleration would substantially reduce these errors, so they have been neglected in computing the pre-injection errors.

When the MSFN is used to determine the pre-injection position and velocity using updating, the venting acceleration is estimated by the MSFN, assuming the magnitude of the venting acceleration is constant. This results in slightly optimistic estimates of the pre-injection position and velocity errors.

The effects of all these venting approximations become even less important when the injection errors are examined because of the additional errors caused by the injection boost.

The position and velocity errors prior to injection are listed in Table 1. The computations leading to these errors are given in the Appendix. The errors for the MSFC and MIT systems are based on 3σ errors in the navigation systems, while those for the MSFN correspond to the accuracies stated in 3, 4 and 5, above. 3σ values are used for the on-board systems because of MSC's desire to demonstrate that the Apollo mission can be accomplished even if the on-board system does not perform according to the published specifications. 3σ errors in the on-board systems are employed, in all cases, in computing the injection errors. These injection errors are listed in Table 2.

The same translunar trajectory has been employed in these calculations whether injection occurs on the first, second or third orbit. This should not greatly affect the results and greatly reduces the amount of calculation necessary.

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Table 1.

Position and Velocity Errors Prior to Injection

Orbit Number	System	x (m) (radial)	y (m) (tangent)	z (m) (perp)	$\dot{x}$ (m/sec)	$\dot{y}$ (m/sec)	$\dot{z}$ (m/sec)
1	MSFC	930	14,500	0	3.2	1.5	3.2
	MIT	20,100	106,000	0	13.9	32.3	13.9
	MSFN 800 m, 6 m/sec	461	461	461	3.5	3.5	3.5
	400 m, 2 m/sec	230	230	230	1.2	1.2	1.2
	1000 m, 10 m/sec	580	580	580	5.8	5.8	5.8
2	MSFC	930	18,500	0	3.2	1.5	3.2
	MIT	20,100	268,000	0	13.9	32.3	13.9
	MSFN 800 m, 6 m/sec	461	461	461	3.5	3.5	3.5
	400 m, 2 m/sec	230	230	230	1.2	1.2	1.2
	100 m, 1 m/sec	58	58	58	0.58	0.58	0.58
3	MSFC	930	24,500	0	3.2	1.5	3.2
	MIT	20,100	444,000	0	13.9	32.3	13.9
	MSFN 800 m, 6 m/sec	461	461	461	3.5	3.5	3.5
	400 m, 2 m/sec	230	230	230	1.2	1.2	1.2
	100 m, 0.2 m/sec	58	58	58	0.12	0.12	0.12

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Table 2.

Position and Velocity Errors at Injection

Orbit Number	System	x (m) (radial)	y (m) (tangent)	z (m) (perp)	$\dot{x}$ (m/sec)	$\dot{y}$ (m/sec)	$\dot{z}$ (m/sec)
1	MSFC	1,520	14,500	1,120	4.4	1.5	4.4
	MIT	20,600	107,000	4,480	14.4	32.4	14.4
	MSFN 800 m, 6 m/sec, MSFC	1,400	1,230	1,400	4.9	3.5	4.9
	MIT	1,420	1,320	1,420	5.0	4.3	5.0
	400 m, 2m/sec, MSFC	792	433	792	3.6	1.2	3.6
	MIT	888	670	822	3.8	2.8	3.8
	1000 m, 10m/sec, MSFC	2,060	1,920	2,060	6.7	5.8	6.7
	MIT	2,080	2,000	2,080	6.8	6.3	6.8
2	MSFC	2,030	18,500	1,800	8.2	1.5	8.2
	MIT	20,600	268,000	4,480	14.4	32.4	14.4
	MSFN 800 m, 6 m/sec, MSFC	1,940	1,230	1,940	8.5	3.5	8.5
	MIT	1,420	1,320	1,420	5.0	4.3	5.0
	400 m, 2m/sec, MSFC	1,440	433	1,440	7.8	1.2	7.8
	MIT	822	670	822	3.8	2.8	3.8
	100 m, 1m/sec, MSFC	1,320	193	1,320	7.7	0.6	7.7
	MIT	680	511	680	3.7	2.6	3.7
3	MSFC	2,670	24,500	2,500	12.6	1.5	12.6
	MIT	20,600	440,000	4,480	14.4	32.4	14.4
	MSFN 800 m, 6m/sec, MSFC	2,550	1,230	2,550	12.6	3.5	12.6
	MIT	1,420	1,320	1,420	5.0	4.3	5.0
	400 m, 2m/sec, MSFC	2,130	433	2,130	12.2	1.2	12.2
	MIT	822	670	822	3.8	2.8	3.8
	100 m, 0.2m/sec, MSFC	1,950	71	1,950	12.2	0.2	12.2
	MIT	602	429	602	3.7	2.6	3.7

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Figures 1, 2 and 3 show the expected value of corrective boost and the standard deviation of this value for injection on the first, second and third orbits respectively. Figures 4, 5 and 6 show the corresponding perilune misses at the time of perilune corresponding to a perfect injection. The velocity errors at this time are not shown; prior Bissett-Berman Apollo notes have shown these velocity errors to be small.

As in previous Apollo notes, the guidance rule is that the commanded corrective boost shall reduce the expected value of error in position at the scheduled time of perilune for the reference mission to zero.

The misses at perilune after the first corrective boost are largely independent of the number of the orbit on which injection occurs. They are also largely independent of the navigation system employed, with the exception of the MIT inertial system, which used alone results in misses about an order of magnitude larger.

Among the eight systems considered, that which requires the smallest corrective boost varies with the number of the orbit during which injection occurs, and, in some cases upon the time at which the corrective boost is performed. The results are summarized in Table 3.

As in previous notes, there is a substantial saving in corrective boost costs if the first corrective boost is performed early, and the cost of subsequent corrective boosts is small.

Table 3.

Ranking of Systems According to Required Corrective Boost

Injection:	Orbit 1		Orbit 2		Orbit 3	
Correction:	Early	Late	Early	Late	Early	Late
High Boost	2	2	2	2	2	2
	8	8	1	3	1	3
	7	7	3	4	3	1
	4	4	4	1	5	4
	3	3	5	6	4, 7	5
	1	1, 6	6, 7	8, 5		6
	6				6	7
Low Boost	5	5	8	7	8	8

System	
1	MSFC only
2	MIT only
3	MSFN 800 m, 6 m/sec, MSFC
4	MSFN 800 m, 6 m/sec, MIT
5	MSFN, 400 m, 2m/sec, MSFC
6	MSFN, 400 m, 2m/sec, MIT
7	MSFN, updating, MSFC
8	MSFN, updating, MIT

[redacted]

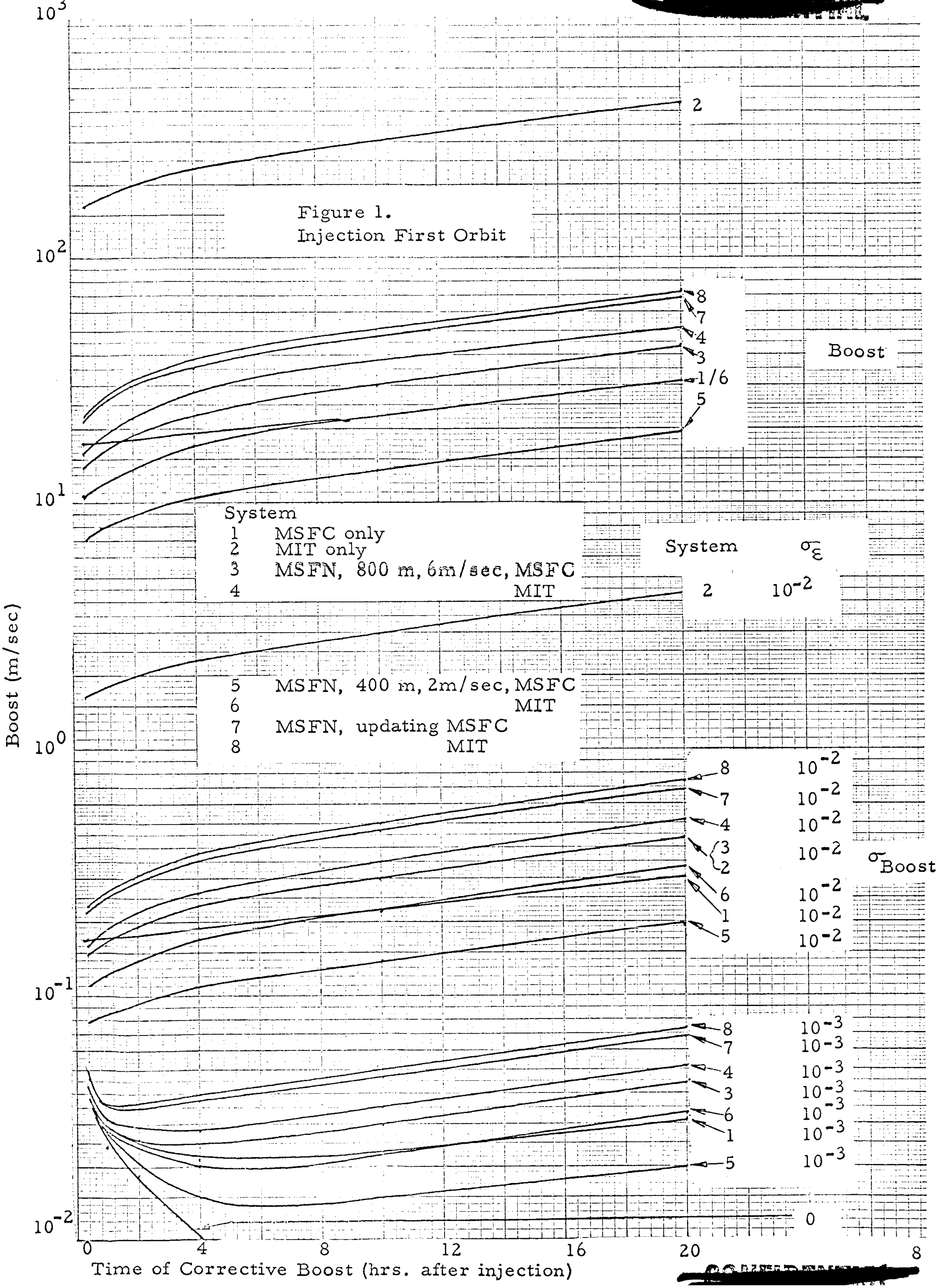

Figure 1.
Injection First Orbit

[redacted]

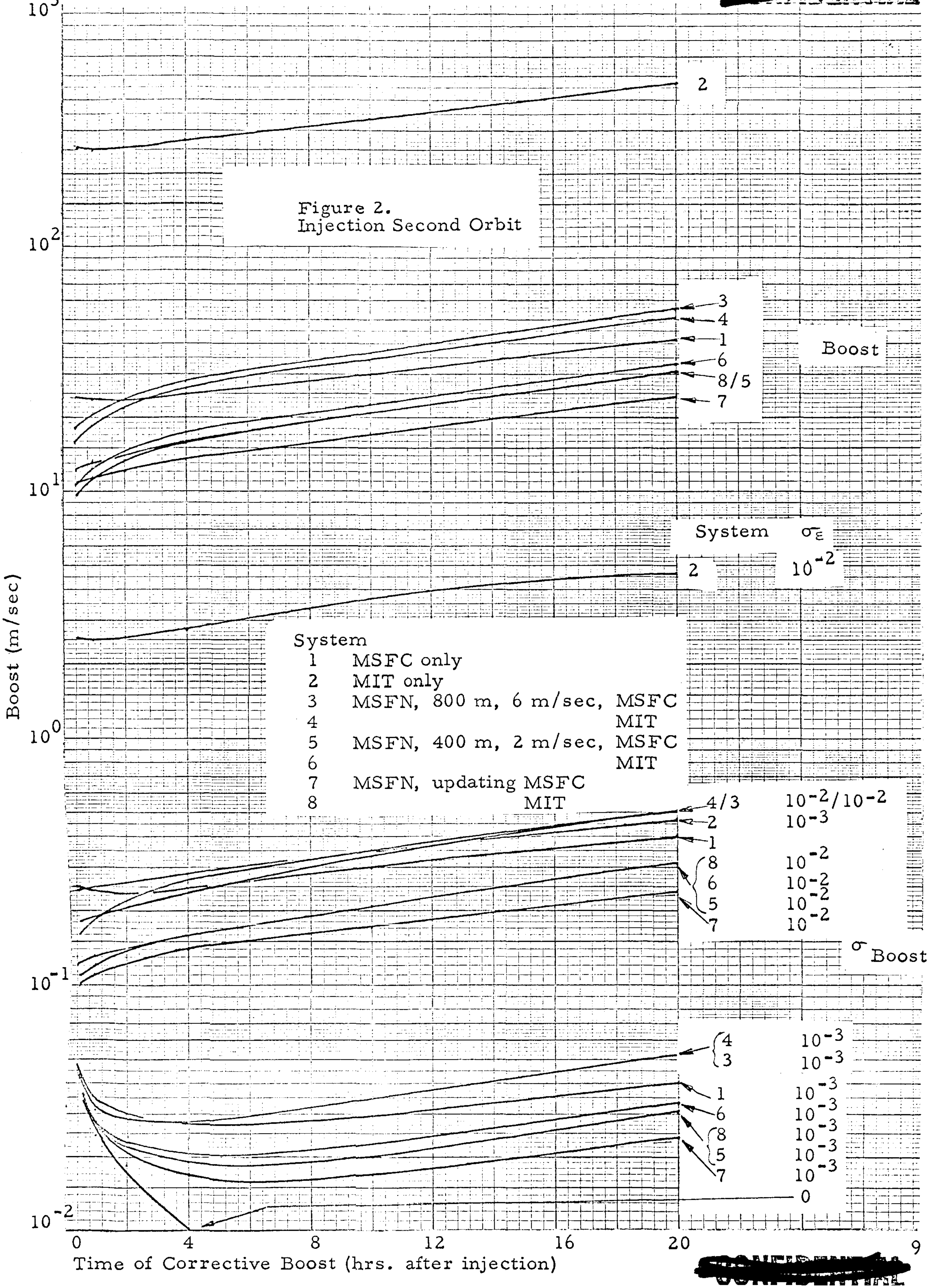

Figure 2.
Injection Second Orbit

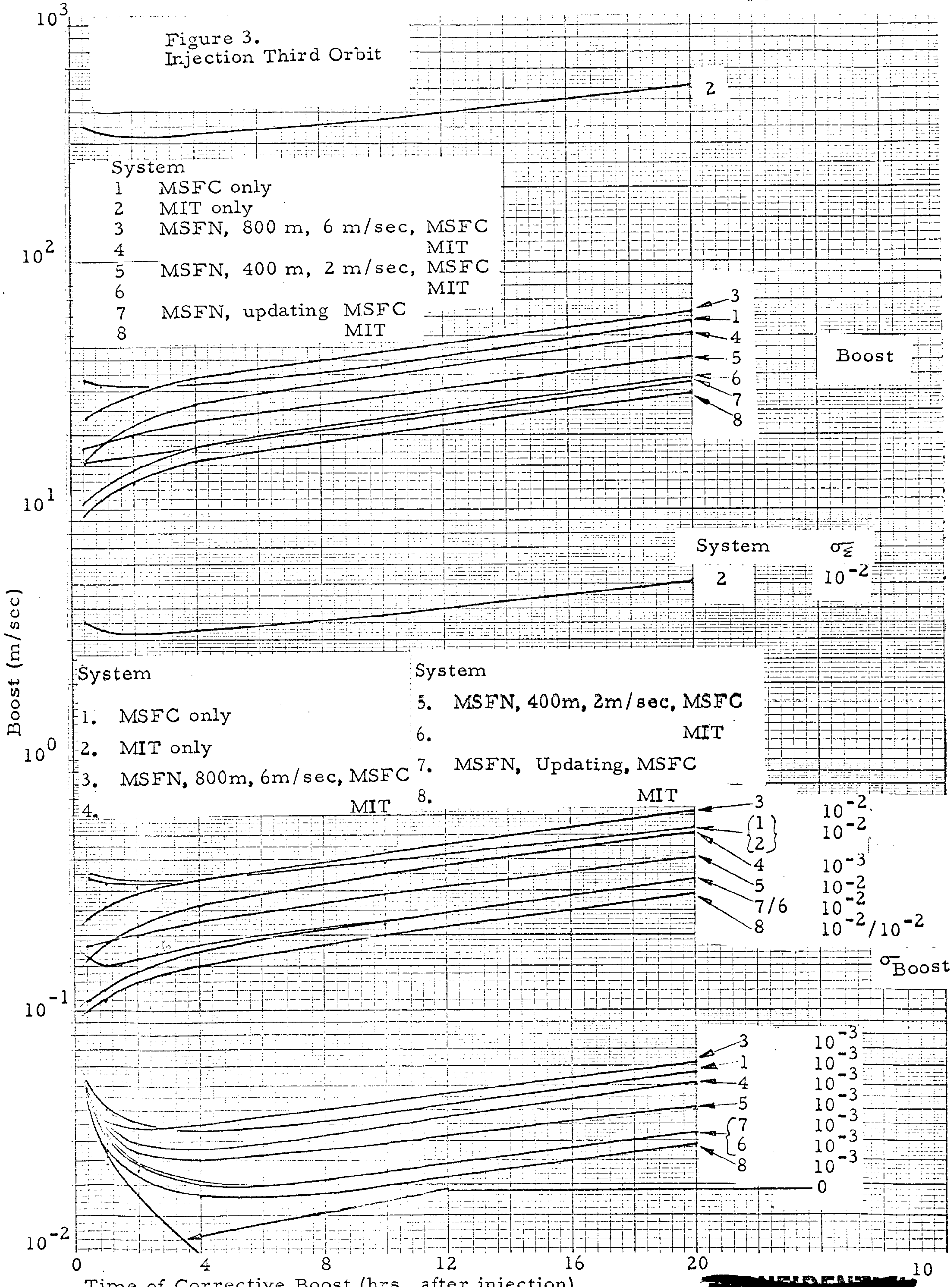

Figure 3.
Injection Third Orbit
System
1 MSFC only
2 MIT only
3 MSFN, 800 m, 6 m/sec, MSFC
4 MIT
5 MSFN, 400 m, 2 m/sec, MSFC
6 MIT
7 MSFN, updating MSFC
8 MIT
Boost
System σ_ε
2 10^{-2}
System
1. MSFC only
2. MIT only
3. MSFN, 800m, 6m/sec, MSFC
4. MIT
5. MSFN, 400m, 2m/sec, MSFC
6. MIT
7. MSFN, Updating, MSFC
8. MIT
σ_{Boost}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
Boost (m/sec)
0 4 8 12 16 20
Time of Corrective Boost (hrs. after injection)

10

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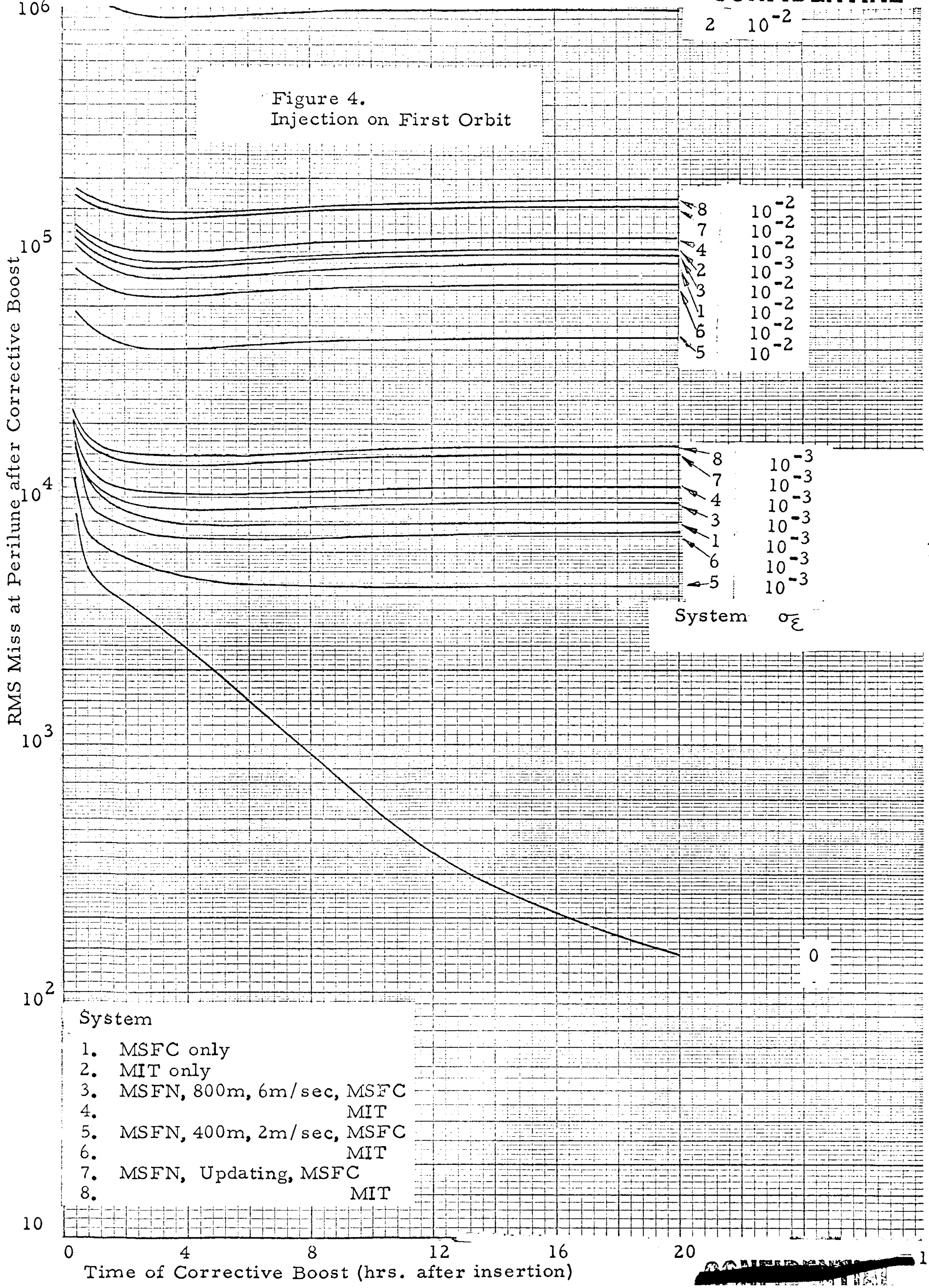
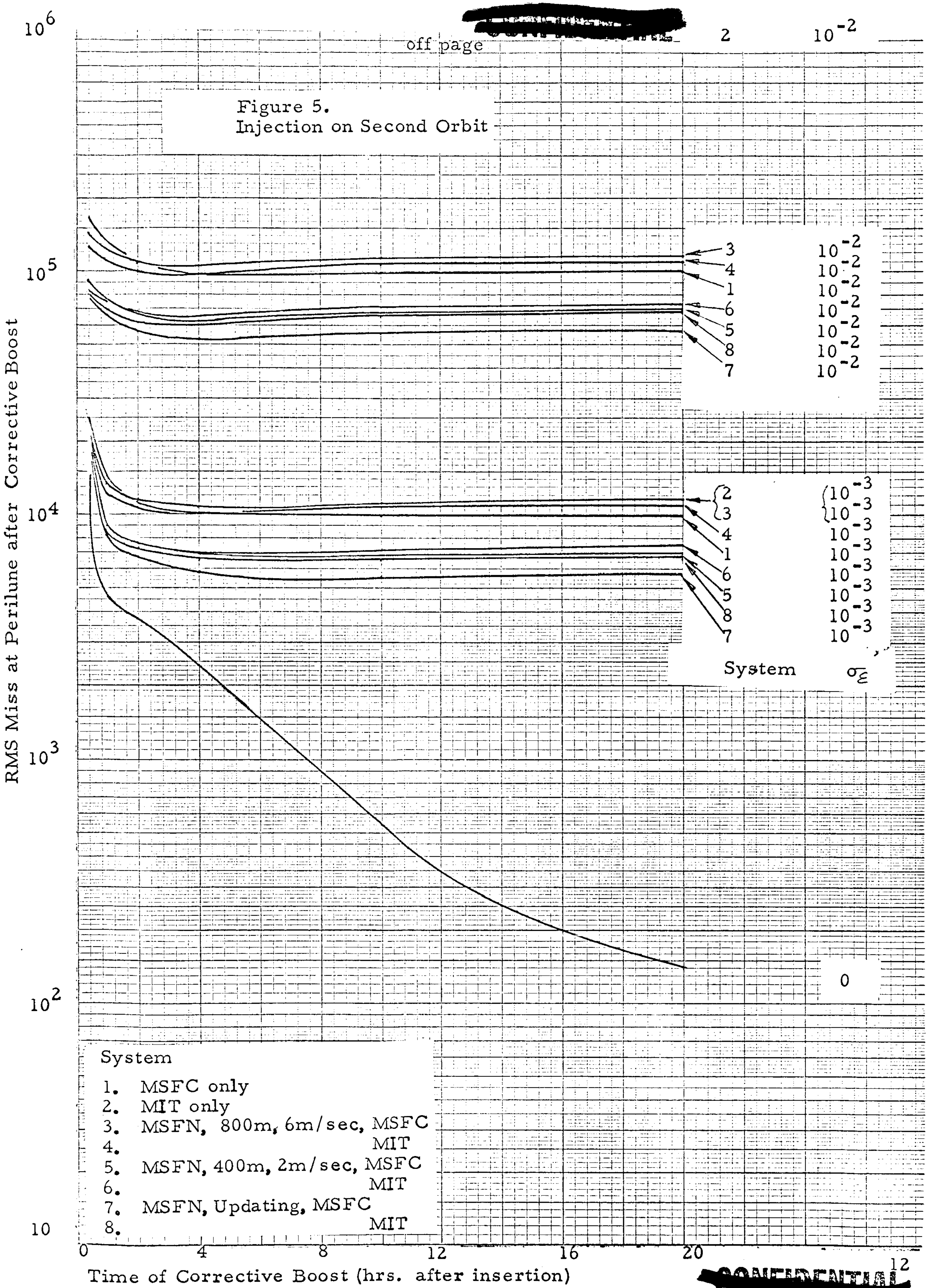


Figure 4.
Injection on First Orbit

[redacted]



off page
2
10^{-2}
Figure 5.
Injection on Second Orbit
10^6
10^5
10^4
10^3
10^2
10
RMS Miss at Perilune after Corrective Boost
3 4 1 6 5 8 7
10^{-2} 10^{-2} 10^{-2} 10^{-2} 10^{-2} 10^{-2} 10^{-2}
2 3 4 1 6 5 8 7
10^{-3} 10^{-3} 10^{-3} 10^{-3} 10^{-3} 10^{-3} 10^{-3} 10^{-3} 10^{-3}
System σ_ε
0
System
1. MSFC only
2. MIT only
3. MSFN, 800m, 6m/sec, MSFC
4. MIT
5. MSFN, 400m, 2m/sec, MSFC
6. MIT
7. MSFN, Updating, MSFC
8. MIT
0 4 8 12 16 20
Time of Corrective Boost (hrs. after insertion)

[redacted]

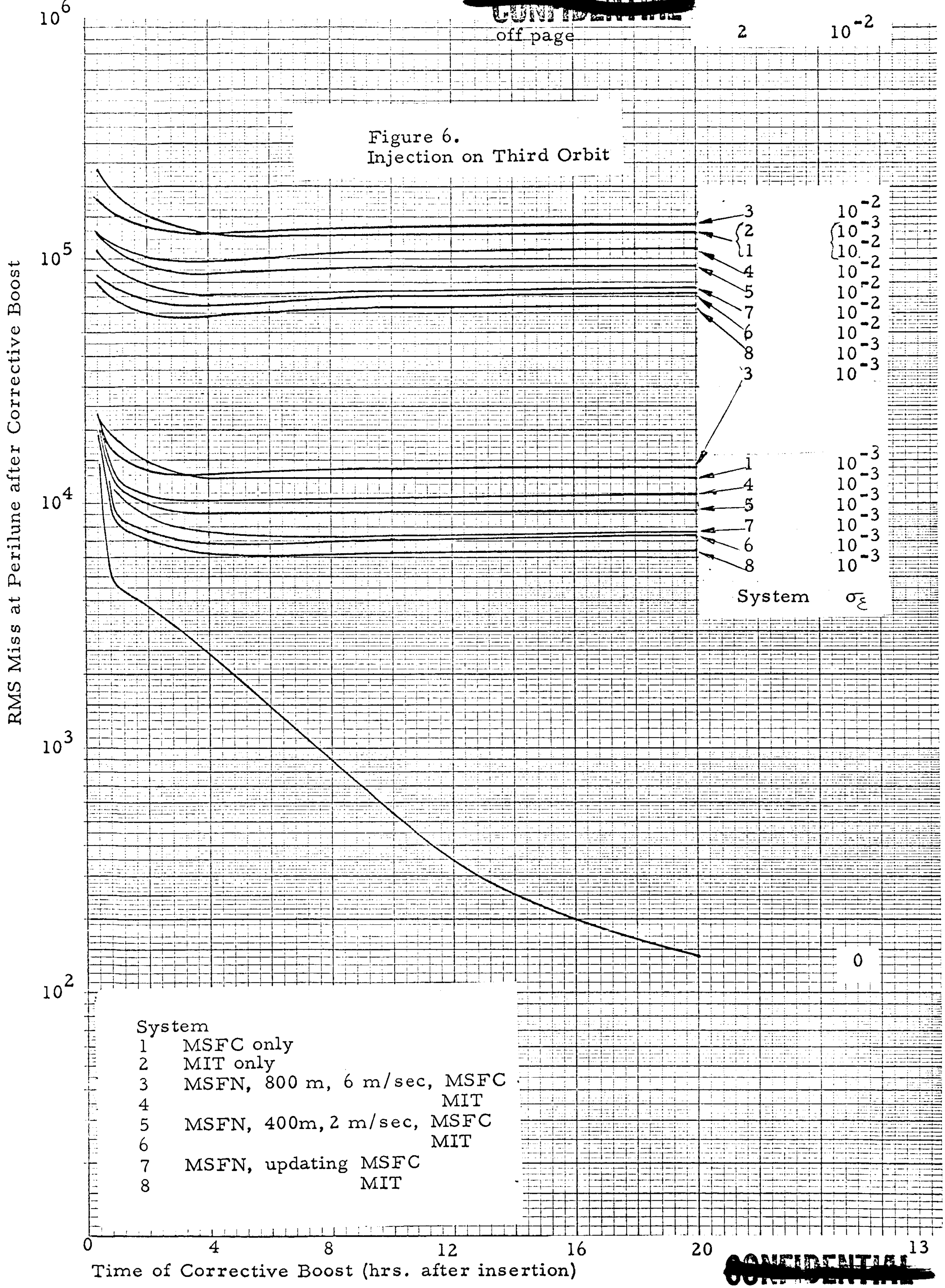


Figure 6.
Injection on Third Orbit

[redacted]

[redacted]

APPENDIX

H. Dale
23 October 1964

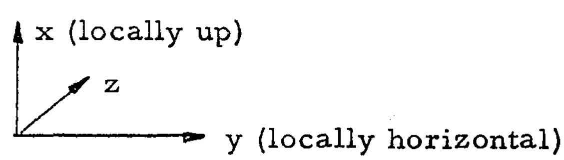
POSITION AND VELOCITY ERRORS AFTER INJECTION FOR VARIANCE ASSUMPTIONS

Two inertial platforms are studied: a) the MIT IMU and b) the MSFC system. Tracking by the MSFN C-band radars during the parking orbit is assumed to give various degrees of uncertainty in the position and velocity just before the injection boost. All of this data may be combined to give the expected uncertainty in position and velocity at the end of the injection boost. The numbers used are 3 σ for the platforms and 1 σ for the MSFN.

<u>Assumed Platform Characteristics (3 σ)</u>

	MSFC		MIT IMU*
initial misalignment in each axis	$=.01^{\circ}$	,	$.035^{\circ}$
drift due to the square of acceleration	$=.05^{\circ}/\mathrm{hr}/g^2$	,	$.045^{\circ}/\mathrm{hr}/g^2$
drift due to the acceleration	$=.05^{\circ}/\mathrm{hr}/g$	,	$.675^{\circ}/\mathrm{hr}/g$
drift bias	$=.05^{\circ}/\mathrm{hr}$	,	$.225^{\circ}/\mathrm{hr}$
accelerometer bias	$=.000324\ \mathrm{ft/sec}^2$	,	$.01965\ \mathrm{ft/sec}^2$
accelerometer axis orthogonality error	$=.0014^{\circ}$	,	$.017^{\circ}$
accelerometer scale factor error	$= 2 \times 10^{-5}\ g/g$	,	$3 \times 10^{-4}\ g/g$
accelerometer error due to acc. square	= 0 (not given)	,	$3 \times 10^{-5}\ g/g^2$

<u>Assumed Trajectory</u>



Boosts are all assumed to be horizontal.

* The MIT IMU (but not the MSFC system) can be aligned to $.035^{\circ}$ before injection.

[redacted]

Boost Segment	1	2	3	4	Insertion Total	Injection
Δtime (min.)	2.50	0.58	5.84	2.81	11.73	5.28
Δ Boost (ft/sec.)	11,300	470	14,400	3,400	29,970	10,380
a (ft/sec.^{2})	75.5	13.4	41.2	20.2		32.7
a (g's)	2.34	.416	1.28	.628		1.02

The trajectory of the vehicle contains a parking orbit between insertion and injection of 1/2, 1-1/2, or 2-1/2 orbits about the Earth. The elapsed time of these orbits is taken to be 45, 135, or 225 minutes.

Assumed MSFN Characteristics

Case 1: The MSFN is not used at all

Case 2: The MSFN increases its ability with respect to tracking time such that the RMS (spherically distributed) position and velocity errors are as given in Apollo Note No. 253, pages 9, 10:

	1/2 orbits	1-1/2 orbits	2-1/2 orbits
RMS position uncertainty	1000 m	100 m	100 m
RMS velocity uncertainty	10 m/s	1 m/s	.2 m/s

Case 3: The MSFN has the same RMS position and velocity uncertainty on any orbit, 800 m and 6 m/s.

Case 4: As above with 400 m and 2 m/s.

The x, y and z component errors are equal to the above errors divided by the square root of three. These numbers differ from those used in previous notes by this factor.

To simplify the analysis it will be assumed that both boosts are straight and locally horizontal (in the y direction). The gyro errors tend to cause equal expected errors in x and z by rotating the total velocity gained away from the y axis. Bias and non-orthogonality errors associated with the accelerometers tend to cause x and z errors also, while bias and scale factor errors should combine to cause velocity errors in the y direction. The insertion boost may be split into thrust segments and the attitude and velocity errors calculated for each segment. Position errors during insertion will be neglected since they are small relative to position error caused by the velocity errors acting over the parking orbit. This is done in tabular form on the following page.

Now velocity errors propagate with time according to the small perturbation equations of Apollo Note No. 7. For the special case of 1/2 orbit, 1-1/2 orbits and 2-1/2 orbits:

$$\sigma\,\dot{x}\ \left(\text{at angle } \theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}\right) = \sigma\,\dot{x}_o$$

$$\sigma\,\dot{y} = 7\,\sigma\,\dot{y}_o$$

$$\sigma\,\dot{z} = \sigma\,\dot{z}_o$$

$$\sigma\,x = 4\,\sigma\,\dot{y}_o/\omega$$

$$\sigma\,y = \sqrt{(4\,\sigma\,\dot{x}_o/\omega)^2 + (3\,\sigma\,\dot{y}\ \frac{\theta}{\omega})^2}$$

$$\sigma\,z = 0$$

where $\omega = 1.16 \times 10^{-3}$ rad/sec. = orbital angular rate, and it is assumed that σx_o, σy_o, and σz_o are neglected. This allows the errors generated by the insertion boost to be accounted for just prior to injection.

Attitude and Velocity Uncertainty After Injection

	MSFC						MIT IMU					
Boost Segment	0	1	2	3	4	Total	0	1	2	3	4	Total
Delta time (min)	0	2.5	0.58	5.84	2.81	11.73	0	2.5	0.58	5.84	2.81	11.73
Boost (ft/sec)	0	11,300	470	14,400	3,400	29,970	0	11,300	470	14,400	3,400	29,970
Gyro a^2 drift (°)	0	.01145	.00008	.00795	.00093	.02041	0	.0103	-	.00715	.00084	.0184
Gyro a drift (°)	0	.00488	.00020	.00620	.00148	.01276	0	.0605	-	.0837	.0200	.1725
Gyro bias drift (°)	0	.00208	.00049	.00491	.00234	.00982	0	.0097	-	.0221	.0101	.0443
Initial Angle Error (°)	.01					.01°	.035					.035
Total Cumulative RMS Angle	.01	.01608	.01628	.02574	.02785	.02785	.035	.0712	.0715	.1120	.1145	.1145
$\dot{x}, \dot{z}$ error (ft/sec) due to gyro	0	2.57	.13	5.28	1.59	9.57	0	10.47	.58	23.06	6.73	40.84
$\dot{x}, \dot{z}$ error (ft/sec) due to accelerometer						.76						8.99
$\dot{y}$ error (ft/sec) due to gyro						.0008						-
$\dot{y}$ error (ft/sec) due to accelerometer						.64						13.83
Total $\dot{x}, \dot{z}$ error (ft/sec)						9.6						41.8
Total $\dot{y}$ error (ft/sec)						.64						13.83

Without MSFN Help

Position and Velocity Errors

Just Prior to Injection (meters, meters/second)

	MSFC			MIT IMU		
	1/2 orbit	1-1/2 orbits	2-1/2 orbits	1/2 orbit	1-1/2 orbits	2-1/2 orbits
$\dot{x}$	3.2	3.2	3.2	13.9	13.9	13.9
$\dot{y}$	1.5	1.5	1.5	32.3	32.3	32.3
$\dot{z}$	3.2	3.2	3.2	13.9	13.9	13.9
x	930	930	930	20,100	20,100	20,100
y	14,500	18,500	24,500	106,000	268,000	440,000
z	0	0	0	0	0	0

The angle error of the platform half-way through the injection boost may be calculated as before and then used to find the velocity errors made due to the platform angular errors during injection. It is assumed that the MSFC system drifts from the time of lift-off while the MIT IMU can be aligned just prior to the injection boost.

Platform Errors Made During Injection

MSFC	MIT IMU
a^2 gyro drift = .0204 + .05 (.044) 1.04 = .0227°	.045 (.044) 1.04 = .0021°
a gyro drift = .0128 + .05 (.044) 1.02 = .0150	.675 (.044) 1.02 = .0303
gyro bias = (2-1/2 orbits) = .1995	.225 (.044) = .0099
(1-1/2 orbits = .1245	(same for all orbits
(1/2 orbit) = .0490	since updated)
initial alignment = .01	= .0344
Total RMS angle 1/2 way (2-1/2 orbits) = .2016°	= .0465°
through boost (1-1/2 orbits) = .1278	
(1/2 orbit) = .0569	
$\dot{x}, \dot{z}$ error (ft/sec) (2-1/2 orbits) = 36.49	8.41
due to gyro = (1-1/2 orbits) = 23.13	
$\frac{\text{Total Angle}}{5.73}$ (10,380) (1/2 orbit) = 10.30	
$\dot{x}, \dot{z}$ error (ft/sec) due to accelerometer bias and non-orthoginality = .28	7.0
$\dot{y}$ error (ft/sec) due to accelerometer bias, scale factor and a^2 error = .28	7.7

It is now possible to combine the errors made during the injection boost with the various assumed sets of position and velocity errors depending upon the MSFN assumptions previously discussed. This is done in an RMS sense with position uncertainty calculated by taking the square root of the sum of two squares, the first being the MSFN estimate of position prior to the boost, and the second being the boost time multiplied by the mean velocity uncertainty during the boost. For the cases with no MSFN help, the uncertainty prior to the injection boost is that given by the guidance systems alone (which appears in a previously shown table).

Final Position and Velocity Uncertainties
After Injection with No MSFN Help
(meters and meters/second)

	MSFC			MIT IMU*		
	1/2 orbit	1-1/2 orbits	2-1/2 orbits	1/2 orbit	1-1/2 orbits	2-1/2 orbits
$\dot{x}$	4.37	8.18	12.57	14.38	14.38	14.38
$\dot{y}$	1.50	1.50	1.50	32.40	32.40	32.40
$\dot{z}$	4.37	8.18	12.57	14.38	14.38	14.38
x	1518	2030	2667	20,590	20,590	20,590
y	14,507	18,505	24,503	106,500	268,000	440,000
z	1120	1804	2500	4480	4480	4480

* The MIT IMU is assumed to be aligned in angle ($3\sigma = .035^{\circ}$) just prior to the injection boost.

Final Position and Velocity Uncertainties After
Injection with Bissett-Berman Assumed Capabilities
of the MSFN (as reported in Apollo Note 253) (meters, meters/second)

	MSFC			MIT IMU		
	1/2 orbit	1-1/2 orbits	2-1/2 orbits	1/2 orbit	1-1/2 orbits	2-1/2 orbits
$\dot{x}$	6.72	7.73	12.2	6.82	3.70	3.66
$\dot{y}$	5.77	.58	.15	6.31	2.63	2.56
$\dot{z}$	6.72	7.73	12.2	6.82	3.70	3.66
x	2060	1320	1950	2080	680	602
y	1920	193	71.3	2000	512	429
z	2060	1320	1950	2080	680	602

Final Position and Velocity Uncertainties After Injection with MSFN Capabilities Described as 800 m and 6 m/s RMS

(meters and meters/second)

	MSFC			MIT IMU		
	1/2 orbit	1-1/2 orbits	2-1/2 orbits	1/2 orbit	1-1/2 orbits	2-1/2 orbits
$\dot{x}$	4.88	8.45	12.6	5.02	same	same
$\dot{y}$	3.47	3.47	3.47	4.31		
$\dot{z}$	4.88	8.45	12.6	5.02		
x	1400	1940	2550	1420		
y	1230	1230	1230	1320		
z	1400	1940	2550	1420		

Final Position and Velocity Uncertainties After Injection with MSFN Capabilities Described as 400 m and 2 m/s RMS

(meters and meters/second)

	MSFC			MIT IMU		
	1/2 orbit	1-1/2 orbits	2-1/2 orbits	1/2 orbit	1-1/2 orbits	2-1/2 orbits
$\dot{x}$	3.62	7.80	12.2	3.82	same	same
$\dot{y}$	1.16	1.16	1.16	2.81		
$\dot{z}$	3.62	7.80	12.2	3.82		
x	792	1440	2130	822		
y	433	433	433	670		
z	792	1440	2130	822		